2.3

Intermediate Value Theorem (IVT)

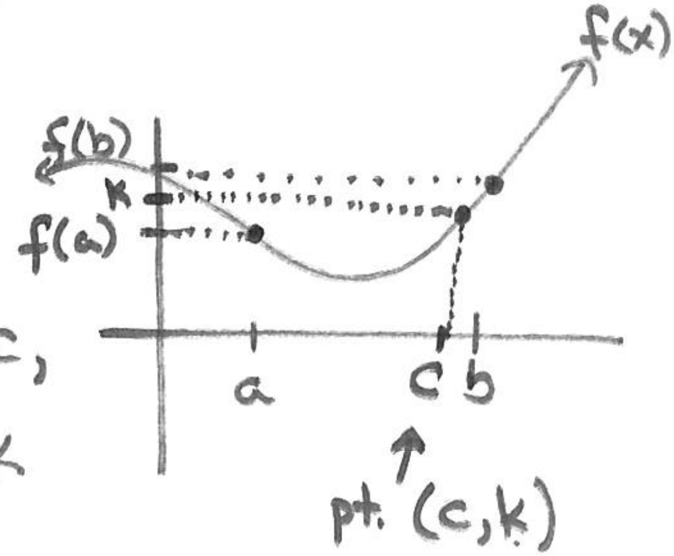
If f is cont on $[a,b]$

and K is any # btn $f(a)$ and $f(b)$,

then $\exists$ at least one #, c, on $[a,b]$ s.t. $f(c)=K$

there exists

such that



conditions needed:

① f cont on $[a,b]$

② K (y-value) btn $f(a)$ + $f(b)$

conclusion:

$\exists$ a c s.t. $f(c)=K$

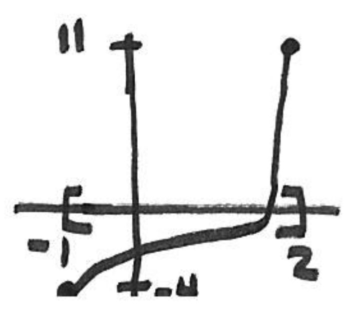
ex: Use IVT to show $f(x)=x^3+2x-1$ has a zero on $[-1,2]$

K

a

b

① f cont? yes b/c $f(x)$ is polynomial



③ $f(-1) = (-1)^3 + 2(-1) - 1 = -4$
$f(2) = 2^3 + 2(2) - 1 = 11$ $\Big\}$ zero is btn. -4 and 11

Since f cont on $[-1, 2]$ and

$$\underbrace{f(-1) < 0 \text{ and } f(2) > 0}_{f(-1) < 0 < f(2)}$$

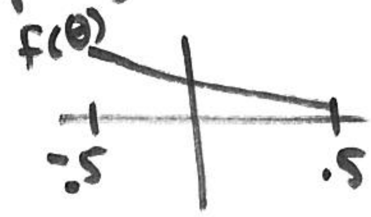
~~tho~~ $\therefore \exists$ a zero on $[-1, 2]$

f1(x)

ex: $f(\theta) = 1 + \theta - 3\tan\theta$ on $\left[\overset{a}{-\tfrac{1}{2}}, \overset{b}{\tfrac{1}{2}}\right]$.

Does IVT guarantee that $f(c) = \overset{k}{1}$?

$f(\theta)$ cont? yes b/c graphing calc



$f(-\frac{1}{2}) = 2.139$
$f(\frac{1}{2}) = -.139$ $\Big\}$ 1 is btn 2.139 & $-.139$

Since $f(\theta)$ cont on $[-\frac{1}{2}, \frac{1}{2}]$ and

$f(-\frac{1}{2}) > 1$ and $f(\frac{1}{2}) < 1$

$f(\frac{1}{2}) < 1 < f(-\frac{1}{2})$.

$\therefore \exists$ a c, on $[-\frac{1}{2}, \frac{1}{2}]$ s.t. $f(c) = 1$

ex: $f(x) = \dfrac{x-1}{x+2}$ on $[-5, 5]$

Does IVT guarantee that $f(c) = 0$?

f cont? no, @ $x = -2$ $\exists$ a V.A.

IVT cannot be used.

ex: $f(x) = x^2 + x - 1$ on $[\overset{a}{0}, \overset{b}{4}]$

Does IVT guarantee that $f(c) = \underset{k}{20}$?

f cont? yes b/c polynomial

$f(0) = 0^2 + 0 - 1 = -1$

$f(4) = 4^2 + 4 - 1 = 19$

Since $f(0) < 20$ and $f(4) < 20$,

IVT cannot be used.