DATE: __________

2.4 Rates of Change & Limits – Examples

Ex. **1:** Find the slope of the tangent line to the graph of $f(x) = x^2 - 3x$ at (1, -2).

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$f'(x) = \lim_{h \to 0} \frac{(x+h)^2 - 3(x+h) - (x^2 - 3x)}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{x^2} + 2xh + h^2 - \cancel{3x} - 3h - \cancel{x^2} + \cancel{3x}}{h}$$

$$= \lim_{h \to 0} \frac{2xh + h^2 - 3h}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{h}(2x + h - 3)}{\cancel{h}}$$

$$= \lim_{h \to 0} (2x + h - 3)$$

$$f'(x) = 2x - 3$$

$$f'(1) = 2(1) - 3 = \boxed{-1}$$

Ex. **2:** Find the slope of the normal line to the graph of $f(x) = x^2 - 3x$ at (1, -2).

↳ $\perp$ to tangent line,

so since $f'(1) = -1$

slope of normal line = opposite reciprocal

$$= -\frac{1}{-1} = \boxed{1}$$

Ex. 3: Find the equation of the tangent line and the normal line to the graph of $f(x) = x^2 + 2x + 1$ at (-3, 4).
Then, use your graphing calculator to check your answer.

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{(x+h)^2 + 2(x+h) + 1 - (x^2 + 2x + 1)}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{x^2} + 2xh + h^2 + \cancel{2x} + 2h + \cancel{1} - \cancel{x^2} - \cancel{2x} - \cancel{1}}{h}$$

$$= \lim_{h \to 0} \frac{2xh + h^2 + 2h}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{h}(2x + h + 2)}{\cancel{h}}$$

$$= \lim_{h \to 0} (2x + h + 2)$$

$$f'(x) = 2x + 2$$

$$f'(-3) = 2(-3) + 2 = -4 \leftarrow \text{slope}$$

$$y - y_1 = m(x - x_1)$$

$$\boxed{y - 4 = -4(x - {}^-3) \text{ tangent Line}}$$

$$\boxed{y - 4 = \tfrac{1}{4}(x - {}^-3) \text{ normal Line}}$$