2.4 Rates of Change + Limits

$$\text{avg rate of change} = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$$

ex: Find avg rate of change $f(x) = x^2 - 3$ on the interval $[-1, 2]$

x_1 x_2

$$\text{avg rate of change} = \frac{f(2) - f(-1)}{2 - -1}$$

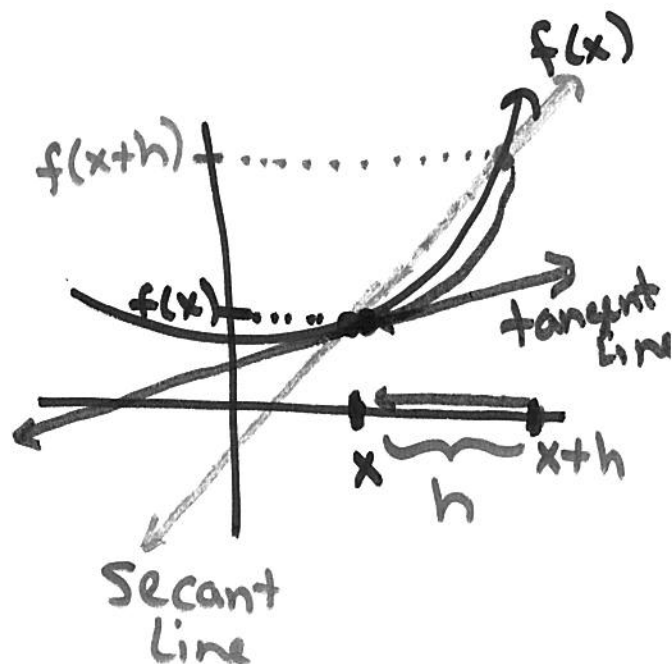$$\boxed{\frac{f(b) - f(a)}{b - a}}$$

$$= 1$$

· Tangent Lines

To write an equation of Line, slope + pt.

$$y - y_1 = m(x - x_1)$$

$$\text{slope} = \frac{y_2 - y_1}{x_2 - x_1}$$

pts. $(x, f(x))$ and $(x+h, f(x+h))$

$$m = \frac{f(x+h) - f(x)}{x+h - x}$$

$$= \frac{f(x+h) - f(x)}{h}$$ ← slope for secant line

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$ ← slope of tangent line

"f prime of x"

2.4 Rates of Change & Limits – Examples

Ex. 1: Find the slope of the tangent line to the graph of $f(x) = x^2 - 3x$ at $(1, -2)$.

$\uparrow$ pt

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{(x+h)^2 - 3(x+h) - (x^2 - 3x)}{h}$$

$(x+h)(x+h)$

$$= \lim_{h \to 0} \frac{x^2 + 2xh + h^2 - 3x - 3h - x^2 + 3x}{h}$$

$$= \lim_{h \to 0} \frac{2xh + h^2 - 3h}{h}$$

$$= \lim_{h \to 0} \frac{h(2x + h - 3)}{h}$$

$$= \lim_{h \to 0} (2x + h - 3)$$

$f'(x) = 2x - 3$ ← general slope

$f'(1) = 2(1) - 3 = \boxed{-1}$ ← slope of tangent @ $x = 1$

Ex. 2: Find the slope of the normal line to the graph of $f(x) = x^2 - 3x$ at $(1, -3)$.