DATE: _______________

Tangent Line Problems

1) For $f(x) = x^2 - 8x + 9$,

a) find the average rate of change on [2,3].

$f(3) = 3^2 - 8(3) + 9 = -6$

$f(2) = 2^2 - 8(2) + 9 = -3$

$$\text{avg rate of change} = \frac{f(3) - f(2)}{3-2} = \frac{-6-(-3)}{1} = \boxed{-3}$$

b) find the instantaneous rate of change at $x = 3$.

$\lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$

$f(x) = x^2 - 8x + 9$

$f(3+h) = (3+h)^2 - 8(3+h) + 9$

$$f'(3) = \lim_{h\to 0} \frac{f(3+h) - f(3)}{h}$$
$$= \lim_{h\to 0} \frac{(3+h)^2 - 8(3+h) + 9 - (-6)}{h}$$
$$= \lim_{h\to 0} \frac{9 + 6h + h^2 - 24 - 8h + 9 + 6}{h}$$
$$= \lim_{h\to 0} \frac{-2h + h^2}{h}$$
$$= \lim_{h\to 0} \frac{h(-2+h)}{h} = \lim_{h\to 0} (-2+h) = \boxed{-2}$$

2) Find the equation of the line tangent to $g(x) = x^3 + x$ at (2,10).

$y - y_1 = m(x - x_1)$

$(2+h)^3 = 2^3 + 3(2)^2h + 3(2)h^2 + h^3$

$$g'(2) = \lim_{h\to 0} \frac{g(2+h) - g(2)}{h}$$
$$= \lim_{h\to 0} \frac{(2+h)^3 + (2+h) - 10}{h}$$
$$= \lim_{h\to 0} \frac{8 + 6h^2 + 12h + h^3 + 2 + h - 10}{h}$$
$$= \lim_{h\to 0} \frac{6h^2 + 13h + h^3}{h}$$
$$= \lim_{h\to 0} \frac{\cancel{h}(6h + 13 + h^2)}{\cancel{h}} = \lim_{h\to 0} (6h + 13 + h^3) = 13$$

$$\boxed{y - 10 = 13(x - 2)}$$

3) Derive the formula for the slope of the curve of $h(x) = \frac{1}{1-x}$.

$$h'(x) = \lim_{h\to 0} \frac{\frac{1}{1-(x+h)} - \frac{1}{1-x}}{h}$$
$$= \lim_{h\to 0} \frac{\frac{1-x}{1-x}\cdot\frac{1}{1-x-h} - \frac{1}{1-x}\cdot\frac{1-x-h}{1-x-h}}{h}$$
$$= \lim_{h\to 0} \frac{\frac{1-x-(1-x-h)}{(1-x)(1-x-h)}}{h}$$
$$= \lim_{h\to 0} \frac{1-x-1+x+h}{(1-x)(1-x-h)} \cdot \frac{1}{h}$$
$$= \lim_{h\to 0} \frac{\cancel{h}}{(1-x)(1-x-h)} \cdot \frac{1}{\cancel{h}}$$
$$= \lim_{h\to 0} \frac{1}{(1-x)(1-x+h)}$$
$$\boxed{h'(x) = \frac{1}{(1-x)^2}}$$