DATE: _____

Instantaneous Speed, Rate of Change, & Slopes of Tangent Lines

1. A rock breaks loose from the top of a tall cliff. The position function for the rock is $f(t) = 16t^2$ feet. Find the speed of this falling rock at 3 seconds.

↳ instantaneous speed

$$\text{inst speed} = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$$

$$= \lim_{h\to 0} \frac{16(x+h)^2 - 16x^2}{h}$$

$$= \lim_{h\to 0} \frac{16(x^2+2xh+h^2) - 16x^2}{h}$$

$$= \lim_{h\to 0} \frac{16x^2+32xh+16h^2-16x^2}{h}$$

$$= \lim_{h\to 0} \frac{32xh+16h^2}{h}$$

$$= \lim_{h\to 0} \frac{h(32x+16h)}{h}$$

$$= \lim_{h\to 0} (32x+16h)$$

$$= 32x$$

inst speed @ 3 sec = 32(3)
= 96 ft/sec

2. Evaluate the rate of change of the area of a circle when $r = 7$ feet.

↳ inst. rate of change

$A = \pi r^2$

$$\text{inst rate of change} = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$$

$$= \lim_{h\to 0} \frac{\pi(x+h)^2 - \pi x^2}{h}$$

$$= \lim_{h\to 0} \frac{\pi(x^2+2xh+h^2) - \pi x^2}{h}$$

$$= \lim_{h\to 0} \frac{\pi x^2+2\pi xh+\pi h^2-\pi x^2}{h}$$

$$= \lim_{h\to 0} \frac{2\pi xh+\pi h^2}{h}$$

$$= \lim_{h\to 0} \frac{h(2\pi x+\pi h)}{h}$$

$$= \lim_{h\to 0} (2\pi x+\pi h)$$

$$= 2\pi x$$

inst rate of change @ $r=7 \Rightarrow 2\pi(7)$
14π

3. At what point is the tangent to $f(x) = 3 - 4x - x^2$ horizontal?
(HINT: What is the slope of a horizontal line?)

↳ slope is zero

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h\to 0} \frac{3 - 4(x+h) - (x+h)^2 - (3 - 4x - x^2)}{h}$$

$$= \lim_{h\to 0} \frac{3 - 4x - 4h - (x^2 + 2xh + h^2) - 3 + 4x + x^2}{h}$$

$$= \lim_{h\to 0} \frac{-4h - x^2 - 2xh - h^2 + x^2}{h}$$

$$= \lim_{h\to 0} \frac{h(-4 - 2x - h)}{h}$$

$$= \lim_{h\to 0} (-4 - 2x - h)$$

$$0 = -4 - 2x$$

$$4 = -2x$$

$$\boxed{-2 = x}$$

4. A line is tangent to a graph $y = f(x)$ at the point $(1, 4)$ and the tangent line passes through $(3, 6)$. What is $f'(1)$?

↳ slope of tangent line

$$f'(1) = \frac{4 - 6}{1 - 3}$$

$$= \frac{-2}{-2}$$

$$= \boxed{1}$$