2.4 Real Zeroes of Polynomial Functions

Target 2C: Find Real and Complex Zeroes of Polynomials by Synthetic and Long Division

SAT Connection

Passport to Advanced Mathematics

11. Understand the relationship between zeros and factors of polynomials.

Example: For a polynomial $p(x)$, the value of $p(3)$ is -2. Which of the following must be true about $p(x)$?

A) $x-5$ is a factor of $p(x)$.

B) $x-2$ is a factor of $p(x)$.

C) $x+2$ is a factor of $p(x)$.

D) The remainder when $p(x)$ is divided by $x-3$ is -2.

$p(3) = -2$
Remainder theorem states: "if a polynomial $f(x)$ is divided by $x-K$, then $f(K) = \text{remainder}$"
$K=3$
remainder $= -2$

Solution

Rational Zeroes Theorem

Watch a video or view a website to learn about Rational Zeroes Theorem
http://www.wtamu.edu/academic/anns/mps/math/mathlab/col_algebra/col_alg_tut38_zero1.htm
https://www.youtube.com/watch?v=7p2yeuAXSCs

Given a polynomial with integer coefficients,

$$f(x) = \underbrace{a_n}_{\text{factors of L.C. are called } q} x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + \underbrace{a_0}_{\text{factors of constant are called } p},$$

then $x = \frac{p}{q}$ is a rational zero of $f(x)$.

where $\frac{p}{q} = \dfrac{\text{factors of the constant}}{\text{factors of the leading coefficient}}$

(write an example from the website/video)

Example 1:

examples vary by student

Example 2:

Find the rational zeroes of $f(x) = 2x^3 - 3x^2 - 23x + 12$

- Factors of the constant → $p = \{\pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 12\}$
- Factors of the *l.c.* → $q = \{\pm 1, \pm 2\}$
- Possible rational zeroes: $\frac{p}{q} = \{\pm 1, \pm \frac{1}{2}, \pm 2, \pm 3, \pm \frac{3}{2}, \pm 4, \pm 6, \pm 12\}$

* choose a possible rational zero & do synthetic division or remainder theorem *

by synthetic division

$$\begin{array}{r|rrrr} 2 & 2 & -3 & -23 & 12 \\ & & 4 & 2 & -42 \\ \hline & 2 & 1 & -21 & -30 \end{array}$$

← remainder not zero, so $x = 2$ is not a zero try another #

$$\begin{array}{r|rrrr} 1 & 2 & -3 & -23 & 12 \\ & & 2 & -1 & -24 \\ \hline & 2 & -1 & -24 & -12 \end{array}$$

∴ $x = 1$ is not a zero

$$\begin{array}{r|rrrr} 4 & 2 & -3 & -23 & 12 \\ & \downarrow & 8 & 20 & 12 \\ \hline & 2 & 5 & -3 & 0 \end{array}$$

← ∴ $x = 4$ is a zero $x = 4$, $x - 4 = 0$ … ☺

$f(x) = (2x^2 + 5x - 3)(x - 4)$

$= (2x - 1)(x + 3)(x - 4)$

∴ $\boxed{\text{the zeroes are: } \tfrac{1}{2}, 4, -3}$

OR

by remainder theorem

$F(2) = 2(2)^3 - 3(2)^2 - 23(2) + 12$
$= -30$ ☹

$f(1) = 2(1)^3 - 3(1)^2 - 23(1) + 12$
$= -12$ ☹

$f(\tfrac{1}{2}) = 2(\tfrac{1}{2})^3 - 3(\tfrac{1}{2})^2 - 23(\tfrac{1}{2}) + 12$
$= 0$ ☺

$f(4) = 2(4)^3 - 3(4)^2 - 23(4) + 12$
$= 0$ ☺

$f(-3) = 2(-3)^3 - 3(-3)^2 - 23(-3) + 12$

$\boxed{\text{the zeroes are: } \tfrac{1}{2}, 4, -3}$

Example 3:
Find the zeroes of $f(x) = x^3 - 6x^2 + 7x + 4$ and identify as rational or irrational.

Factors of constant: $p = \{\pm 1, \pm 4, \pm 2\}$

Factors of L.C.: $q = \{\pm 1\}$

$p/q = \{\pm 1, \pm 4, \pm 2\}$

used remainder theorem:

$f(1) = 1^3 - 6(1)^2 + 7(1) + 4$
$= 6$:(

$f(-1) = (-1)^3 - 6(-1)^2 + 7(-1) + 4$
$= -10$:(

$f(4) = 4^3 - 6(4)^2 + 7(4) + 4$
$= 0$:)

$$\begin{array}{r|rrrr} 4 & 1 & -6 & 7 & 4 \\ & & 4 & -8 & -4 \\ \hline & 1 & -2 & -1 & 0 \end{array}$$

$f(x) = (x^2 - 2x - 1)(x - 4)$

$\boxed{\text{rational zero: } x = 4}$

$x^2 - 2x - 1$ is not factorable, so use quadratic formula

$a = 1$, $b = -2$, $c = -1$

$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x = \dfrac{2 \pm \sqrt{(-2)^2 - 4(1)(-1)}}{2(1)}$

$x = \dfrac{2 \pm \sqrt{8}}{2}$

$x = \dfrac{2 \pm 2\sqrt{2}}{2}$

$x = \dfrac{2}{2} \pm \dfrac{2\sqrt{2}}{2}$

$\boxed{x = 1 \pm \sqrt{2} \text{ irrational zeroes}}$

More Practice

Rational Zeroes Theorem

http://www.sparknotes.com/math/algebra2/polynomials/section4.rhtml
http://www.virtualnerd.com/algebra-2/polynomials/roots-zeros/rational-zero-theorem/rational-zeros-example
http://www.math-prof.com/Alg2/Alg2_Ch_16.asp
https://www.youtube.com/watch?v=YMyv9-9VXw4
https://www.youtube.com/watch?v=7mNBBBspqUc

Homework Assignment

p.217 #33,34,49,51,54,71,72

SAT Connection

Solution

Choice D is correct. If the polynomial $p(x)$ is divided by $x - 3$, the result can be written as $\frac{p(x)}{x - 3} = q(x) + \frac{r}{x - 3}$, where $q(x)$ is a polynomial and r is the remainder. Since $x - 3$ is a degree 1 polynomial, the remainder is a real number. Hence, $p(x)$ can be written as $p(x) = (x - 3)q(x) + r$, where r is a real number. It is given that $p(3) = -2$ so it must be true that $-2 = p(3) = (3 - 3)q(3) + r = (0)q(3) + r = r$. Therefore, the remainder when $p(x)$ is divided by $x - 3$ is -2.

Choice A is incorrect because $p(3) = -2$ does not imply that $p(5) = 0$. Choices B and C are incorrect because the remainder -2 or its negative, 2, need not be a root of $p(x)$.