DATE:

### 3.1 Definition of a Derivative

Slope of a Function at a Point


a) Using the graph of $f(x)$ given above, find the slope of $f(x)$ a $x=-3$.
$m=0$
b) Using the graph of $f(x)$ given above, find the slope of $f(x)$ at $x=0$.

$$
m=\frac{-2}{1}=-2
$$

$$
m=\frac{1}{1}=-1 ? \quad m=\frac{3}{1}=\cdot 3 ?
$$

d) Using the graph of $f(x)$ given above, estimate the slope of $f(x)$ at $x=6$.


Visualization at Desmos: https://www.desmos.com/calculator/8ubngtz3ei
Given function, $f(x)=x^{2}-4 \underset{\sim}{x}+4$, find the slope of the function at the point $(3,1)$.

where the $2^{\text {nd }}$ point is $h$ units away from $x=3$.



Moving the $2^{\text {nd }}$ point closer to $(3,1)$ makes $h$ get closer to zero.

$$
n \rightarrow 0
$$

$\lim _{h \rightarrow 0}(h+2)$

$$
\begin{aligned}
& =0+2 \\
& =2
\end{aligned}
$$

and the slope of the function at the point is the same as the slope of the line tangent to the function at that point.

$$
m_{\text {tagntin }}=\frac{2}{1}=2
$$



$$
f^{\prime}(x)=\lim _{h \rightarrow 0} \frac{f(x+h)-f(x)}{h}
$$

- slope of a tangent line to a function/curve
- slope of a function/curve
- slope of a function/curve at a point
- instantaneous rate of change

| Notation for the Derivative | Read As |
| :---: | :---: |
| $f^{\prime}(x)$ | $f$ prime of $x$ |
| $y^{\prime}(x)$ or $\quad y^{\prime}$ | $y$ prime of $x$ or $y$ prime |
| $\frac{d}{d x}(f(x))$ | derivative of $f(x)$ with respect to $x$ |
| $\frac{d y}{d x}$ | derivative of $y$ with respect to $x$ |

Example T: Find the derivative of $f(x)=\left(x^{2}-4(x)+4\right.$
$f^{\prime}(x)=\lim f(x+h)-f(x)=(x+h)^{2}-4(x+h)+4$

$$
\begin{aligned}
f^{\prime}(x) & =\lim _{h \rightarrow 0} \frac{f(x+h)-f(x)}{h} \\
& =\lim _{h \rightarrow 0} \frac{\left(x+h^{2}-4(x+h)+4-\left(x^{2}-4 x+4\right)\right.}{h} \\
& =\lim _{h \rightarrow 0} \frac{\left.x^{2}+2 \cdot h+h^{2}-4 x-4 h+4\right)=(x+h)^{2}-4(x+h)+4}{h} \\
& =\lim _{h \rightarrow 0} \frac{2 x h+x^{2}+4 x-4}{h} \\
& =\lim _{h \rightarrow 0} \frac{h(2 x+h-4)}{h} \\
& =\lim _{h \rightarrow 0}(2 x+h-4) \\
& =2 x+0-4 \\
f^{\prime}(x) & =2 x-4
\end{aligned}
$$

Example 2: A train is traveling back and forth on an east-west section of a railroad track. The train's position, measured in feet, is given by the function, $x(t)=5 t^{2}-3 t$. Find the instantaneous rate of change of this train at 2 seconds.

$$
\begin{aligned}
x^{\prime}(t) & =\lim _{h \rightarrow 0} \frac{x(t+h)-x(t)}{h} \\
& =\lim _{n \rightarrow 0} \frac{5(t+h)^{2}-3(t+h)-\left(5 t^{2}-3 t\right)}{h} \\
& =\lim _{h \rightarrow 0} \frac{5\left(t^{2}+2 t h+h^{2}\right)-3 t-3 h-5 t^{2}+3 t}{h} \\
& =\lim _{h \rightarrow 0} \frac{5 t^{2}+10 t h+5 h^{2}-3 t-3 h-5 t^{2}+3 t}{h} \\
& =\lim _{h \rightarrow 0} \frac{10 t h+5 h^{2}-3 h}{h} \\
& =\lim _{h \rightarrow 0} \frac{x(10 t+5 h-3)}{h}=\lim _{n \rightarrow 0}(10 t+5 h-3)=10 t-3
\end{aligned}
$$

Alternate Form of the Derivative

The slope of a function, $f(x)$, at a given point, $x=a$, is:

$$
f^{\prime}(a)=\lim _{x \rightarrow a} \frac{y_{2}(x)-f(a)}{\substack{y_{1} \\ x-a}}
$$




Need to do or see more practice?
Go to https://www.mathkanection.com/bc-unit-2-derivatives.html\#definitionderivative

