

## 2.6 Complex Numbers & the Fundamental Theorem of Algebra

Target 2D: Construct Polynomials given Real and/or Imaginary Zeros

Target 2E: Understand the Fundamental Theorem of Algebra

### Review of Prior Concepts

Solve each quadratic equation:

a)  $x^2 = -4$

$$\begin{aligned}\sqrt{x^2} &= \sqrt{-4} \\ x &= \pm \sqrt{-4} \\ x &= \pm 2i\end{aligned}$$

b)  $x^2 + 5 = 4x$

$$\begin{aligned}x^2 - 4x + 5 &= 0 \\ x &= \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(5)}}{2(1)} \\ &= \frac{4 \pm \sqrt{16 - 20}}{2} \\ &= \frac{4 \pm \sqrt{-4}}{2} \\ &= \frac{4 \pm 2i}{2} \rightarrow \frac{4}{2} \pm \frac{2i}{2} = 2 \pm i\end{aligned}$$

### More Practice

#### Complex Solutions

<http://www.regentsprep.org/regents/math/algtrig/ate3/quadcomlesson.htm>

<http://www.coolmath.com/algebra/10-complex-numbers/03-quadratic-formula-01>

<https://www.mathsisfun.com/numbers/complex-numbers.html>



#### SAT Connection

#### Passport to Advanced Math

4. Create an equivalent form of an algebraic expression

Example: Which of the following complex numbers is

equivalent to  $\frac{3-5i}{8+2i}$ ? (Note:  $i = \sqrt{-1}$ )

A)  $\frac{3}{8} - \frac{5i}{2}$

$$\begin{aligned}\frac{3-5i}{8+2i} \cdot \frac{8-2i}{8-2i} &= \frac{24-6i-40i+10i^2}{64-4i^2} \\ &= \frac{24-46i-10}{64+4} \\ &= \frac{14-46i}{68} \\ &= \frac{14}{68} - \frac{46i}{68} \\ &= \frac{7}{34} - \frac{23}{34}i\end{aligned}$$

B)  $\frac{3}{8} + \frac{5i}{2}$

C)  $\frac{7}{34} - \frac{23i}{34}$

D)  $\frac{7}{34} + \frac{23i}{34}$

#### Solution

## Fundamental Theorem of Algebra



Vocabulary Term	In my own words...	Example(s)
Fundamental Theorem of Algebra	A polynomial function, $f(x)$ with degree $> 0$ has # of complex zeroes = degree (some zeros can repeat)	$f(x) = (x-2)(x+3)$ degree : 2 # of zeroes : 2 $g(x) = (x-2)^2(x+3)$ degree : 3 # of zeroes : 3 (one zero repeats)

Examples:

Using your graphing calculator, find the complex zeroes and write the polynomial in factored form:

1)  $f(x) = 4x + 3$

degree : 1  
zero : -0.75

$f(x) = 4(x + .75)$

2)  $g(x) = x^2 - 4$

degree : 2  
zeroes : -2, 2

$g(x) = (x+2)(x-2)$

3)  $h(x) = 2x^3 + 3x^2 - 11x - 6$

degree : 3  
zeroes : -3, -0.5, 2

$x = -3$	$x = -0.5$	$x = 2$
$x+3=0$	$x=-\frac{1}{2}$	$x-2=0$
$x+\frac{1}{2}=0$		
$2x+1=0$		

$h(x) = (x+3)(2x+1)(x-2)$

4)  $f(x) = x^3 - 2x^2 + x - 2$

degree : 3  
zeroes : 2, i, -i

synthetic division w/  
zero from graph calc

$$\begin{array}{r}
 2 | 1 & -2 & 1 & -2 \\
 \downarrow & 2 & 0 & 2 \\
 \hline
 1 & 0 & 1 & 0
 \end{array}$$

$\therefore$

$$\begin{aligned}
 x^2 + 1 &= 0 \\
 \sqrt{x^2} &= \sqrt{-1} \\
 x &= \pm i
 \end{aligned}$$

$f(x) = (x-2)(x-i)(x+i)$

## Complex Conjugate Zeros

If  $a + bi$  is a zero of  $f(x)$ , then  $a - bi$  is also a zero of  $f(x)$

\*\*\*\*Imaginary Zeros are always conjugate pairs \*\*\*\*

Examples:

Write a polynomial in standard form with the following zeroes:

1) 4,  $2i$

$$\begin{array}{l} \hookrightarrow \text{also, } -2i \\ \begin{array}{lll} x=4 & x=2i & x=-2i \\ x-4=0 & x-2i=0 & x+2i=0 \end{array} \end{array}$$

$$\begin{aligned} f(x) &= (x-4)(x-2i)(x+2i) \quad \text{multiply conjugate pairs} \\ &= (x-4)(x^2+2ix-2ix-4i^2) \\ &= (x-4)(x^2-4(-1)) \\ &= (x-4)(x^2+4) \end{aligned}$$

$$f(x) = x^3 - 4x^2 + 4x - 16$$

2) 3 (multiplicity 2),  $1-i$  (multiplicity 1)

$$\begin{array}{l} \hookrightarrow \text{also, } 1+i \\ \begin{array}{lll} x=3 & x=1-i & x=1+i \\ x-3=0 & x-1+i=0 & x-1-i=0 \end{array} \end{array}$$

$$\begin{aligned} f(x) &= (x-3)^2(x-1+i)(x-1-i) \\ &= (x-3)^2(x^2-x-ix-x+1+i+ix-i-i^2) \\ &= (x-3)^2(x^2-2x+1-(-1)) \\ &= (x-3)^2(x^2-2x+2) \\ &= (x^2-6x+9)(x^2-2x+2) \\ &= x^4-2x^3+2x^2-6x^3+12x^2-12x+9x^2-18x \\ &\quad + 18 \end{aligned}$$

$$f(x) = x^4 - 8x^3 + 23x^2 - 30x + 18$$

7) Given  $f(x) = x^4 - 2x^3 + 5x^2 + 10x - 50$  has a zero of  $1 + 3i$ . Find all of the zeroes and write a linear factorization of  $f(x)$ .

Synthetic division

$$\begin{array}{r} \underline{1+3i} | \begin{array}{rrrrr} 1 & -2 & 5 & 10 & -50 \\ \downarrow & 1+3i & -10 & -5-15i & 50 \\ \hline \underline{1-3i} | \begin{array}{rrrrr} 1 & -1+3i & -5 & 5-15i & 0 \\ \downarrow & 1-3i & 0 & -5+15i & \\ \hline 1 & 0 & -5 & 0 & \end{array} \end{array} \end{array}$$

$$x^2 - 5 = 0$$

$$x^2 = 5$$

$$x = \pm\sqrt{5}$$

$$x = 1+3i$$

$$x-1-3i=0$$

$$x = 1-3i$$

$$x-1+3i=0$$

$$x = \sqrt{5}$$

$$x-\sqrt{5}=0$$

$\hookrightarrow 1-3i$  is another zero

$$\begin{array}{l} \begin{array}{l} \begin{array}{l} (1+3i)(-1+3i) \\ -1+3i-3i+9i^2 \\ -1-9 = -10 \end{array} \\ \begin{array}{l} (1+3i)(5-15i) \\ 5-15i+15i-45i^2 \\ 5+45 = 50 \end{array} \end{array} \end{array}$$

$$f(x) = (x-1-3i)(x-1+3i)(x-\sqrt{5})(x+\sqrt{5})$$

**More Practice****Fundamental Theorem of Algebra**

<https://www.khanacademy.org/math/algebra2/polynomial-functions/fundamental-theorem-of-algebra/v/fundamental-theorem-of-algebra-intro>

<https://www.mathsisfun.com/algebra/fundamental-theorem-algebra.html>

<https://www.youtube.com/watch?v=NZS3T43NBvE>

<https://www.youtube.com/watch?v=PQr0yVq5ysc>

<https://www.youtube.com/watch?v=gyksK76Dg1c>

**Homework Assignment**

p.234 #3,5,9,11,15,17,20,27,31

**SAT Connection****Solution**

Choice C is correct. To perform the division  $\frac{3 - 5i}{8 + 2i}$ , multiply the numerator and denominator of  $\frac{3 - 5i}{8 + 2i}$  by the conjugate of the denominator,  $8 - 2i$ . This gives  $\frac{(3 - 5i)(8 - 2i)}{(8 + 2i)(8 - 2i)} = \frac{24 - 6i - 40i + (-5i)(-2i)}{8^2 - (2i)^2}$ . Since  $i^2 = -1$ , this can be simplified to  $\frac{24 - 6i - 40i - 10}{64 + 4} = \frac{14 - 46i}{68}$ , which then simplifies to  $\frac{7}{34} - \frac{23i}{34}$ .

Choices A and B are incorrect and may result from misconceptions about fractions. For example,  $\frac{a + b}{c + d}$  is equal to  $\frac{a}{c + d} + \frac{b}{c + d}$ , not  $\frac{a}{c} + \frac{b}{d}$ . Choice D is incorrect and may result from a calculation error.