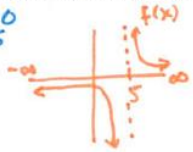
2.7 Rational Functions

Target 2F: Graph, Solve and Analyze Rational Functions

Review of Prior Concepts

Using your graphing calculator, find the domain and describe the end behavior:

a) $f(x) = \frac{1}{x-5}$ $x-5 \neq 0$, $x \neq 5$

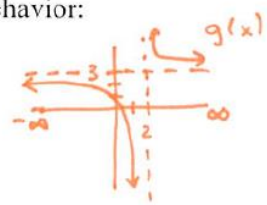


$\lim_{x \to \infty} f(x) = 0$

$\lim_{x \to -\infty} f(x) = 0$

Domain: $(-\infty, -5) \cup (-5, \infty)$

b) $g(x) = \frac{3x-5}{x-2}$ $x-2 \neq 0$, $x \neq 2$



$\lim_{x \to \infty} g(x) = 3$

$\lim_{x \to -\infty} g(x) = 3$

Domain: $(-\infty, 2) \cup (2, \infty)$

More Practice

Domain & End Behavior

http://www.coolmath.com/algebra/15-functions/06-finding-the-domain-01
https://www.khanacademy.org/math/algebra/algebra-functions/domain-and-range/v/domain-of-a-function-intro
https://youtu.be/Krjd_vU4Uvg
https://youtu.be/PQ85Y1jsVzQ

SAT Connection

Passport to Advanced Math

10. Interpret parts of nonlinear expressions in terms of their context

Example:

$$h(x) = \frac{1}{(x-5)^2 + 4(x-5) + 4}$$

For what value of x is the function h above undefined?

h is undefined when denominator = 0

$(x-5)^2 + 4(x-5) + 4 = 0$

$x^2 - 10x + 25 + 4x - 20 + 4 = 0$

$x^2 - 6x + 9 = 0$

$(x-3)^2 = 0$

$x - 3 = 0$

$x = 3$

OR let $y = x - 5$

$y^2 + 4y + 4 = 0$

$(y+2)(y+2) = 0$

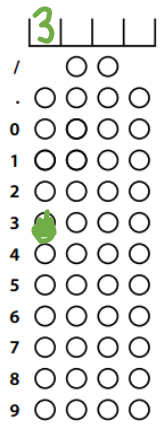
$y = -2$

$y = x - 5$

$-2 = x - 5$

$3 = x$

Solution



NOTE: You may start your answers in any column, space permitting. Columns you don't need to use should be left blank.

Analyzing Graphs of Rational Functions

p. 237

Vocabulary Term	In my own words…	Example(s)
Rational Function	function written as a ratio (fraction) $\frac{f(x)}{g(x)}$ where $f(x)$ + $g(x)$ are polynomials	$h(x)=\frac{3x}{x-2}$ $d(x)=\frac{5x^2+3x-4}{x^2+2}$

Describe the behavior of the graphs of the rational functions at the x-values not in the domain.

a) $f(x)=\frac{1}{x-5}$

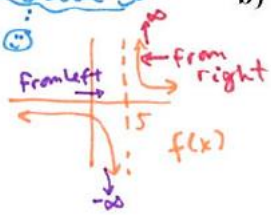
from left 5^-

$\lim_{x\to 5^-} f(x)=-\infty$

from right 5^+

$\lim_{x\to 5^+} f(x)=\infty$

See drawing above

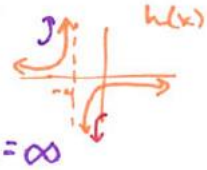


b) $g(x)=\frac{3x-5}{x-2}$

$\lim_{x\to 2^-} g(x)=-\infty$

$\lim_{x\to 2^+} g(x)=\infty$

c) $h(x)=\frac{-5}{x+4}$



$\lim_{x\to -4^-} h(x)=\infty$

$\lim_{x\to -4^+} h(x)=-\infty$

Vertical and Horizontal Asymptotes

Recall:

Vertical asymptotes occur when denominator = 0

Horizontal asymptotes are found from end behavior

Using your graphing calculator, find the vertical and horizontal asymptotes.

a) $f(x)=\frac{3x-5}{x-4}$

V.A. @ $x=4$

H.A. @ $y=3$

b) $g(x)=\frac{3x^2-5}{x^2-4}$

V.A. @ $x=2$, $x=-2$

H.A. @ $y=3$

Without using your graphing calculator, find the vertical and horizontal asymptotes algebraically.

a) $f(x)=\frac{3x-5}{x-4}$

V.A. $x-4=0$

$x=4$

H.A. @ $y=3$

horizontal asymptote

$(x-4)\overline{)3x-5}$

$$\begin{array}{r|rr} 4 & 3 & -5 \\ & \downarrow & 12 \\ \hline & 3 & 7 \end{array}$$

remainder

$f(x)=3+\frac{7}{x-4}$

b) $g(x)=\frac{3x^2-5}{x^2-4}$

V.A. $x^2-4=0$

$(x-2)(x+2)=0$

$x=2,\ x=-2$

H.A. $y=3$

horizontal asymptote

$$\begin{array}{r} 3 \\ x^2-4\overline{)3x^2-5} \\ -(3x^2-12) \\ \hline 7 \end{array}$$

remainder

$g(x)=3+\frac{7}{x^2-4}$

Using your graphing calculator, find the horizontal asymptotes (if any).

a) $f(x) = \frac{3x^2-5x+1}{x^2-4}$

H.A. @ $y = 3$

b) $g(x) = \frac{3x-5}{x^2-4}$

H.A. @ $y = 0$

c) $h(x) = \frac{3x^2-5x+1}{x-4}$

No H.A.

Can you find a pattern? If yes, then find the horizontal asymptotes (if any) without using your graphing calculator.

a) $f(x) = \frac{2x^3+x^2-5x+1}{x^3-4}$

degree of num = degree of denom.

∴, H.A. @ $y = 2$

b) $g(x) = \frac{2x-5}{x^3-4}$

degree of numer < degree of denom.

∴, H.A. @ $y = 0$

c) $h(x) = \frac{2x^3+x^2-5x+1}{x-4}$

degree of numer > degree of denom.

∴, no H.A.

Conclusion about Horizontal Asymptotes:

① If degree of numerator = degree of denominator,
H.A. @ y = leading coefficients

② If degree of numerator < degree of denominator,
H.A. @ $y = 0$

③ If degree of numerator > degree of denominator,
no H.A.

More Practice

Vertical Asymptotes

http://www.sosmath.com/calculus/limcon/limcon04/limcon04.html
https://www.khanacademy.org/math/algebra2/rational-expressions-equations-and-functions/discontinuities-of-rational-functions/v/analyzing-vertical-asymptotes-of-rational-functions
https://www.youtube.com/watch?v=cIkIKyRsybY
https://www.youtube.com/watch?v=ALFNl6QHbVU

Horizontal Asymptotes

http://www.coolmath.com/precalculus-review-calculus-intro/precalculus-algebra/18-rational-functions-finding-horizontal-slant-asymptotes-01
http://www.softschools.com/math/calculus/finding_horizontal_asymptotes_of_rational_functions/
https://www.youtube.com/watch?v=E0iNtii45KA

Homework Assignment

p.247 #1,5,8,9

SAT Connection

Solution

The correct answer is 3. The function $h(x)$ is undefined when the denominator of $\frac{1}{(x-5)^2+4(x-5)+4}$ is equal to zero. The expression $(x-5)^2+4(x-5)+4$ is a perfect square: $(x-5)^2+4(x-5)+4=((x-5)+2)^2$, which can be rewritten as $(x-3)^2$. The expression $(x-3)^2$ is equal to zero if and only if $x=3$. Therefore, the value of x for which $h(x)$ is undefined is 3.