# 2.8 Solving Equations in One Variable

Target 2F: Graph, Solve and Analyze Rational Functions

*Review of Prior Concepts*

Find the least common denominator (LCD) in the following equations

**a)** $\frac{1}{3x} + \frac{5}{9x^2} = \frac{2}{27}$

$\frac{1}{3x} + \frac{5}{3\cdot 3x} = \frac{2}{3\cdot 3\cdot 3}$

$3\cdot 3\cdot 3\cdot x$

$\boxed{\text{LCD: } 27x}$

**b)** $2 - \frac{3}{x+4} = \frac{12}{x^2+4x}$

$2 - \frac{3}{x+4} = \frac{12}{x(x+4)}$

$x(x+4)$

$\boxed{\text{LCD: } x(x+4)}$

**c)** $\frac{3x}{x+2} + \frac{2}{x-1} = \frac{5}{x^2+x-2}$

$\frac{3x}{x+2} + \frac{2}{x-1} = \frac{5}{(x+2)(x-1)}$

$(x+2)(x-1)$

$\boxed{\text{LCD: } (x+2)(x-1)}$

**More Practice**

**Least Common Denominators**

https://www.mathsisfun.com/least-common-denominator.html
http://www.virtualnerd.com/algebra-1/rational-expressions-functions/add-subtract/add-subtract-unlike-denominators/find-least-common-denominator-example
https://www.youtube.com/watch?v=bDIKlHIui1E

**SAT Connection**

**Passport to Advanced Math**

**9.** Rewrite simple rational expressions

Example: If $x > 3$, which of the following is equivalent to $\dfrac{1}{\dfrac{1}{x+2} + \dfrac{1}{x+3}}$ ?

A) $\dfrac{2x+5}{x^2+5x+6}$

B) $\dfrac{x^2+5x+6}{2x+5}$

C) $2x+5$

D) $x^2+5x+6$

$= \dfrac{1}{\frac{(x+3)}{(x+3)}\frac{1}{x+2} + \frac{1}{x+3}\frac{(x+2)}{(x+2)}}$

$= \dfrac{1}{\frac{x+3}{(x+3)(x+2)} + \frac{x+2}{(x+3)(x+2)}}$

$= \dfrac{1}{\frac{2x+5}{(x+3)(x+2)}}$

reciprocal ☺

$= \dfrac{(x+3)(x+2)}{2x+5}$

$= \dfrac{x^2+5x+6}{2x+5}$

OR

$= \dfrac{1}{\left(\frac{1}{x+2} + \frac{1}{x+3}\right)} \frac{(x+3)(x+2)}{(x+3)(x+2)}$

multiply by LCD ☺

$= \dfrac{(x+3)(x+2)}{\frac{(x+3)(x+2)}{x+2} + \frac{(x+3)(x+2)}{x+3}}$

reduce ☺

$= \dfrac{(x+3)(x+2)}{x+3+x+2}$

$= \dfrac{x^2+5x+6}{2x+5}$

Solution

## Solving Rational Equations

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① Multiply by the Least Common Denominator, LCD.
(to eliminate the fractions/rational expressions)

② Solve for *x*.

③ Check for extraneous solutions.

Example

$x + \frac{3}{x} = 4$

LCD: $x$

$x(x) + \frac{3}{x}(x) = 4(x)$

$x^2 + 3 = 4x$

$x^2 - 4x + 3 = 0$

$(x-3)(x-1) = 0$

$\boxed{x = 3 \quad x = 1}$

check: $3 + \frac{3}{3} = 4$ $\quad$ $1 + \frac{3}{1} = 4$

$3 + 1 = 4$ $\quad$ $1 + 3 = 4$

$4 = 4$ ✓ $\quad$ $4 = 4$ ✓

both check... so both $x=1, x=3$ work

*Examples:*

**a)** $\frac{3}{x+1} + \frac{2}{x} = 2$

LCD: $x(x+1)$

$x(x+1)\left(\frac{3}{x+1}\right) + \frac{2}{x}(x)(x+1) = 2(x)(x+1)$

$x(\cancel{x+1})\left(\frac{3}{\cancel{x+1}}\right) + \frac{2}{\cancel{x}}(\cancel{x})(x+1) = 2(x)(x+1)$

$3x + 2x + 2 = 2(x^2 + x)$

$5x + 2 = 2x^2 + 2x$

$0 = 2x^2 - 3x + 2$

$0 = (2x+1)(x-2)$

$\boxed{x = -\frac{1}{2}, \ x = 2}$

check: $\frac{3}{-\frac{1}{2}+1} + \frac{2}{-\frac{1}{2}} = 2$ $\quad$ $\frac{3}{2+1} + \frac{2}{2} = 2$

$\frac{3}{\frac{1}{2}} + 2 \cdot \frac{-2}{1}$ $\quad$ $\frac{3}{3} + 1$

$3 \cdot 2 - 4$ $\quad$ $1 + 1$

$6 - 4$ $\quad$ $2$

$2$ ✓ $\quad$ ✓

**b)** $2 - \frac{3}{x+4} = \frac{12}{x^2+4x}$

LCD: $x(x+4)$

$x(x+4) \cdot 2 - \frac{3}{x+4} x(x+4) = \frac{12}{x(x+4)} \cdot x(x+4)$

$x(x+4) \cdot 2 - \frac{3}{\cancel{x+4}} \cdot x(\cancel{x+4}) = \frac{12}{\cancel{x(x+4)}} \cancel{x(x+4)}$

$(x^2+4x)2 - 3x = 12$

$2x^2 + 8x - 3x = 12$

$2x^2 + 5x = 12$

$2x^2 + 5x - 12 = 0$

$(2x-3)(x+4) = 0$

$\boxed{x = 3/2}$, $x = -4$ extraneous

check: $2 - \frac{3}{3/2 + 4} = \frac{12}{(3/2)^2 + 4(3/2)}$

$\frac{16}{11} = \frac{16}{11}$ ✓

$2 - \frac{3}{-4+4} = \frac{12}{(-4)^2 + 4(-4)}$

$2 - \frac{3}{0} = \frac{12}{0}$

undefined

does not check

**Solve the equation algebraically. Support your answer graphically.**

**1.** $\frac{4x}{x+4}+\frac{3}{x-1}=\frac{15}{x^2+3x-4}$ LCD: $(x+4)(x-1)$

$(x+4)(x-1)\frac{4x}{x+4}+\frac{3}{x-1}(x+4)(x-1)=\frac{15}{(x+4)(x-1)}(x+4)(x-1)$

$\cancel{(x+4)}(x-1)\frac{4x}{\cancel{x+4}}+\frac{3}{\cancel{x-1}}(x+4)\cancel{(x-1)}=\frac{15}{\cancel{(x+4)(x-1)}}\cancel{(x+4)(x-1)}$

$(x-1)(4x)+3(x+4)=15$

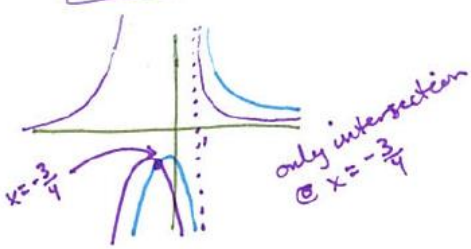
$4x^2-4x+3x+12=15$

$4x^2-x+12=15$

$4x^2-x-3=0$

$(4x+3)(x-1)=0$

$\boxed{x=-3/4}$, $x=1$ extraneous




**2.** $\frac{3}{x+2}+\frac{6}{x^2+2x}=\frac{3-x}{x}$ LCD: $x(x+2)$

$x(x+2)\cdot\frac{3}{x+2}+\frac{6}{x(x+2)}\cdot x(x+2)=\frac{3-x}{x}\cdot x(x+2)$

$x\cancel{(x+2)}\cdot\frac{3}{\cancel{x+2}}+\frac{6}{\cancel{x(x+2)}}\cdot\cancel{x(x+2)}=\frac{3-x}{\cancel{x}}\cdot\cancel{x}(x+2)$

$3x+6=(3-x)(x+2)$

$3x+6=3x+6-x^2-2x$

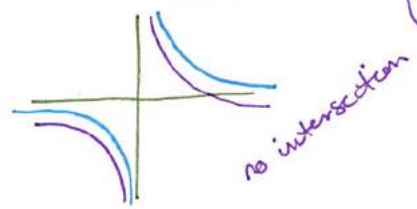
$3x+6=-x^2+x+6$

$x^2+2x=0$

$x(x+2)=0$

$x=0$, $x=-2$ extraneous

$\boxed{\therefore \text{ no solution}}$




## Application of Rational Functions

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*Example:* Consider all rectangles with an area of 182 square feet. Let *x* be the length of one side of such a rectangle.

a) Express the perimeter *P* as a function of *x*.

$x \to$ length $\quad A=lw \quad P=2l+2w$

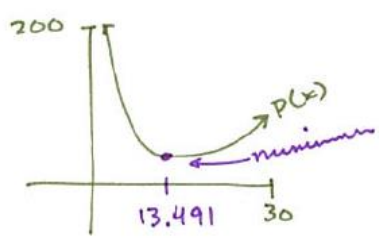
$w \to$ width $\quad 182=x\cdot w \quad P=2x+2w$

$\frac{182}{x}=w$

$P=2x+2\left(\frac{182}{x}\right)$

$P(x)=2x+\frac{364}{x}$

b) Find the dimensions of the rectangle that has the least perimeter. What is the least perimeter?




$x=13.491$

$w=\frac{182}{13.491}=13.491$

$\boxed{\text{rectangle with least perimeter}: 13.491\text{ ft} \times 13.491\text{ ft}}$

least Perimeter : $2(13.491)+2(13.491)$

$=\boxed{53.963\text{ ft}}$

## More Practice

**Solving Rational Equations**

http://www.montereyinstitute.org/courses/Algebra1/COURSE_TEXT_RESOURCE/U11_L2_T1_text_final.html
http://www.regentsprep.org/regents/math/algtrig/ate11/rationalequationsles.htm
http://www.mathplanet.com/education/algebra-1/rational-expressions/solving-rational-expressions
https://www.youtube.com/watch?v=zx82WVQrOCE
https://www.khanacademy.org/math/algebra-home/alg-rational-expr-eq-func/alg-modeling-with-rational-functions/v/applying-rational-equations-1
http://www.mhhe.com/math/devmath/streeter/ia/graphics/streeter5ia/ch07/others/strI_7.6.pdf
https://www.youtube.com/watch?v=-59ijFaVzpU
https://www.youtube.com/watch?v=UcTy7RcZLiw
https://www.youtube.com/watch?v=04TKMJVEsXc

## Homework Assignment

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## SAT Connection

**Solution**

**Choice B is correct.** To rewrite $\dfrac{1}{\dfrac{1}{x+2}+\dfrac{1}{x+3}}$, multiply by $\dfrac{(x+2)(x+3)}{(x+2)(x+3)}$.

This results in the expression $\dfrac{(x+2)(x+3)}{(x+3)+(x+2)}$, which is equivalent to the expression in choice B.

Choices A, C, and D are incorrect and could be the result of common algebraic errors that arise while manipulating a complex fraction.