2008

Section I Answer Key and Percent Answering Correctly
Calculus BC

	Item No.	Correct Answer	Percent Correct by Grade 5	4	3	2	1	Total Percent Correct
	1	B	97	95	93	89	81	93
*	2	A	96	92	87	80	67	89
	3	C	88	77	67	55	36	73
	4	D	95	86	74	61	40	80
	5	C	94	85	76	66	40	80
*	6	A	81	61	52	43	35	63
	7	D	90	73	58	43	23	69
*	8	B	89	71	56	39	26	68
*	9	D	96	85	74	58	33	79
*	10	B	90	80	72	59	39	76
	11	A	75	48	34	27	21	52
	12	D	64	35	25	21	16	42
*	13	B	75	55	40	29	23	55
*	14	E	63	39	32	27	22	45
*	15	A	85	63	45	31	17	61
	16	D	62	38	27	21	16	42
*	17	B	79	60	45	37	28	59
*	18	A	59	33	23	16	13	38
	19	A	61	41	30	23	18	43
	20	C	49	23	16	12	9	30
*	21	A	58	35	23	16	10	38
	22	E	31	11	7	6	5	18
	23	E	80	57	42	28	15	56
	24	A	63	37	25	19	13	41
*	25	B	57	27	19	13	12	35
	26	D	56	35	25	19	13	38
*	27	A	83	66	52	41	28	64
	28	D	26	5	3	2	2	13

* AB Questions

AB Subscore = $\frac{\text{AB pts}}{13/28}$

2008 BC

(1) $a(t) = v'(t)$

$= \langle 2t, 5 \rangle$

$a(3) = \langle 6, 5 \rangle$ $\boxed{B}$

(2) $\int x e^{x^2}\,dx$ $\quad u = x^2$, $\frac{du}{dx} = 2x$, $\frac{du}{2x} = dx$

$\int x e^u \cdot \frac{du}{2x}$

$= \frac{1}{2}\int e^u\,du$

$= \frac{1}{2}e^u + C$

$= \frac{1}{2}e^{x^2} + C$ $\boxed{A}$

(3) $\lim_{x\to 0} \frac{\sin x \cos x}{x}$

$= \left(\lim_{x\to 0}\frac{\sin x}{x}\right)\left(\lim_{x\to 0}\cos x\right)$

$= 1 \cdot \cos 0$

$= 1$

$\boxed{C}$

(4) $\lim_{n\to\infty} \frac{e^{n+1}}{(n+1)!} \cdot \frac{n!}{e^n}$

$\lim_{n\to\infty} \frac{e \cdot n!}{(n+1)!}$

$\lim_{n\to\infty} \frac{e \cdot n!}{(n+1)n!} = \lim_{n\to\infty}\frac{e}{n+1} < 1$

$\boxed{D}$

(5) $L = \int_a^b \sqrt{(x'(t))^2 + (y'(t))^2}\,dt$

$= \int_0^\pi \sqrt{(\cos(t^3)\cdot 3t^2)^2 + (e^{5t}\cdot 5)^2}\,dt$

$= \int_0^\pi \sqrt{9t^4\cos^2(t^3) + 25e^{10t}}\,dt$

$\boxed{C}$

(6) I. $\lim_{x\to 2^-} f(x) = \lim_{x\to 2^+} f(x)$ ✓

II. $\lim_{x\to 2} f(x) = \lim_{x\to 2}\frac{(x-2)(x+2)}{x-2}$ ✗

$= 4$

$f(2) = 1$

$\lim_{x\to 2} f(x) \neq f(2)$ ∴ f not cont @ $x=2$

✗ III. f not cont @ $x=2$,

∴ f not diff'able @ $x=2$

I only $\boxed{A}$

(7)

	Δx	$\frac{dy}{dx}$	$\Delta x \frac{dy}{dx}$	$y + \Delta x \frac{dy}{dx}$
$(1,-3)$	.5	-1	-.5	$-3 + -.5 = -3.5$
$(1.5,-3.5)$	.5	-.5	-.25	$-3.5 + -.25 = -3.75$
$(2,-3.75)$				

$f(2) = -3.75$ $\boxed{D}$

(8) left sum

$\int_2^{13} f(x)\,dx = 1(6) + 2(-2) + 3(-1) + 5(3)$

$= 6 - 4 - 3 + 15$

$= 14$ $\boxed{B}$

(9) greatest → max → g' change from pos to neg.

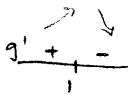
$g'(x) = 0$

$g'(x) = \frac{d}{dx}\int_{-7}^{x} f(t)\,dt$

$g'(x) = f(x)$

$0 = f(x)$

@ $x = 1$



$\boxed{D}$ $g(1)$ max – no need to check endpts b/c $g(-3)$ and $g(2)$ are lower than $g(1)$

(10) $x^2 + xy + y^2 = 7$ @ $(2,1)$

$2x + y(1) + x\frac{dy}{dx} + 2y\frac{dy}{dx} = 0$

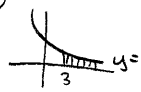
$2(2) + 1 + 2\frac{dy}{dx} + 2\frac{dy}{dx} = 0$

$5 + 4\frac{dy}{dx} = 0$

$4\frac{dy}{dx} = -5$

$\frac{dy}{dx} = -\frac{5}{4}$ $\boxed{B}$

(11)



Area $= \int_3^\infty e^{-2x}\,dx$

$= \lim_{a\to\infty}\int_3^a e^{-2x}\,dx$

$= \lim_{a\to\infty} -\frac{1}{2}e^{-2x}\Big|_3^a = \lim_{a\to\infty} -\frac{1}{2e^{2x}}\Big|_3^a$

$= \lim_{a\to\infty}\left(-\frac{1}{2e^{2a}} - -\frac{1}{2e^6}\right) = \frac{1}{2e^6}$ $\boxed{A}$

(12) (A) $\lim_{n\to\infty}\frac{x^{n+1}}{n+1}\cdot\frac{n}{x^n}$

$= \lim_{n\to\infty} x\left(\frac{n}{n+1}\right)$

$= x$

so, converges on $|x| < 1$

(B) $\lim_{n\to\infty}\frac{x^{n+1}}{(n+1)^2}\cdot\frac{n^2}{x^n}$

$= \lim_{n\to\infty} x\left(\frac{n^2}{(n+1)^2}\right)$

$= x$

so, converges on $|x| < 1$

(C) $\lim_{n\to\infty}\frac{x^{n+1}}{\sqrt{n+1}}\cdot\frac{\sqrt{n}}{x^n}$

$= \lim_{n\to\infty} x\left(\frac{\sqrt{n}}{\sqrt{n+1}}\right)$

$= x$

so, converges on $|x| < 1$

$\boxed{D}$ (D) $\lim_{n\to\infty}\frac{e^{n+1}\cdot x^{n+1}}{(n+1)!}\cdot\frac{n!}{e^n\cdot x^n}$

$= \lim_{n\to\infty} e\cdot x\left(\frac{n!}{(n+1)n!}\right)$

$= 0$, so converges $\forall x$

(E) $\lim_{n\to\infty}\frac{(n+1)!\,x^{n+1}}{e^{n+1}}\cdot\frac{e^n}{n!\,x^n}$

$\lim_{n\to\infty}\frac{x}{e}\left(\frac{(n+1)n!}{n!}\right) = \infty$, diverges

(13) $\int_1^e \frac{x^2+1}{x}\,dx$

$= \int_1^e\left(\frac{x^2}{x} + \frac{1}{x}\right)dx$

$= \int_1^e\left(x + \frac{1}{x}\right)dx$

$= \left(\frac{1}{2}x^2 + \ln|x|\right)\Big|_1^e$

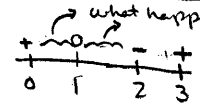
$= \frac{1}{2}e^2 + \ln e - \left(\frac{1}{2}(1)^2 + \ln 1\right)$

$= \frac{1}{2}e^2 + 1 - \frac{1}{2} + 0$

$= \frac{1}{2}e^2 + \frac{1}{2}$

$\boxed{B}$

(14) f''



what happens btn (0,1) and btn (1,2)?

(A) f inc → $f' > 0$
no info about f'

(B) f dec → $f' < 0$
no info about f'

(C) f rel max @ $x=1$, so $f'(1) = 0$ and $f''(1) < 0$
not true, $f''(1) = 0$ and no info about $f'(1)$

(D) inf pt @ $x=1$, f'' changes signs ... maybe, but signs on (0,1) and (1,2) unknown

(E) f changes concavity on (0,2)
yes ... b/c $f''(x) > 0$ @ $x=0$
$f''(x) < 0$ @ $x=2$

$\boxed{E}$

(15) $f(x) = (\ln x)^2$

$f'(x) = 2(\ln x)\cdot\frac{1}{x}$

$= \frac{2}{x}\ln x$

$f''(x) = \ln x(-2x^{-2}) + \frac{2}{x}\left(\frac{1}{x}\right)$

$= -\frac{2\ln x}{x^2} + \frac{2}{x^2}$

$f''(\sqrt{e}) = -\frac{2\ln\sqrt{e}}{(\sqrt{e})^2} + \frac{2}{(\sqrt{e})^2}$

$= \frac{-2\left(\frac{1}{2}\right)}{e} + \frac{2}{e}$

$= -\frac{1}{e} + \frac{2}{e} = \frac{1}{e}$ $\boxed{A}$

(16) $\sum_{n=1}^{\infty} \left(\frac{2}{x^2+1}\right)^n$

geometric converges $|r|<1$

$\left|\frac{2}{x^2+1}\right| < 1$

$2 < |x^2+1|$

$|x^2+1| > 2$

$x^2+1>2 \qquad x^2+1<-2$

$x^2>1 \qquad \cancel{x^2<-1}$

$|x|>1$

$x>1,\ x<-1$ $\boxed{D}$

(17) $f'(x) = h'(x^2-3)\cdot 2x$

$f'(2) = h'(2^2-3)\cdot 2(2)$

$= h'(1)\cdot 4$

$= 4h'(1)$ $\boxed{B}$

(18) $x+y=k$

$y=-x+k$

$m=-1$

$y=x^2+3x+1$

$\frac{dy}{dx} = 2x+3$

$2x+3=-1$

$2x=-4$

$x=-2$

$y=(-2)^2+3(-2)+1$

$=4-6+1$

$y=-1$

$x+y=k$

$-2+-1=k$

$-3=k$ $\boxed{A}$

(19) $\int \frac{7x}{(2x-3)(x+2)}dx$

$\frac{7x}{(2x-3)(x+2)} = \frac{A}{2x-3} + \frac{B}{x+2}$

$7x = Ax+2A+2Bx-3B$

$0=2A-3B \qquad 7=A+2B$

$-14=-2A-4B$

$-14=-7B$

$2=B$

$7=A+2(2)$

$7=A+4$

$3=A$

$\int \frac{3}{2x-3}dx + \int \frac{2}{x+2}dx$

$3\cdot\frac{1}{2}\ln|2x-3| + 2\ln|x+2| + C$

$\boxed{A}$

(20) $1+\ln 2+\frac{(\ln 2)^2}{2!}+\dots\frac{(\ln 2)^n}{n!}+\dots$

$e^x = 1+x+\frac{x^2}{2!}+\dots\frac{x^n}{n!}+\dots$

$e^{\ln 2} = 1+\ln 2+\frac{(\ln 2)^2}{2!}+\dots\frac{(\ln 2)^n}{n!}$

$e^{\ln 2} = 2$ $\boxed{C}$

(21) $v(t)$ inc $\rightarrow$ $a(t)$ positive or $x''(t)$ positive

$x''(t)$ pos when $x(t)$ concave up

x'': $+$ on $(0,2)$, $-$ on $(2,6)$

on $(0,2)$ $\boxed{A}$

(22) $\int_0^1 f(x)g'(x)dx = uv - \int v\,du$

$u=f(x) \qquad dv=g'(x)dx$

$du=f'(x)dx \qquad v=g(x)$

$= f(x)g(x)\Big|_0^1 - \int_0^1 g(x)\cdot f'(x)dx$

$= f(1)g(1) - f(0)g(0) - 5$

$= 4(3) - 2(-4) - 5$

$= 12+8-5$

$= 15$ $\boxed{E}$

(23) $f(x) = x\sin(2x)$

$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} + \dots$

$\sin(2x) = 2x - \frac{(2x)^3}{3!} + \frac{(2x)^5}{5!} + \dots$

$= 2x - \frac{8x^3}{3!} + \frac{32x^5}{5!} + \dots$

$x\sin(2x) = 2x^2 - \frac{8x^4}{3!} + \frac{32x^6}{5!} + \dots$

$\boxed{E}$

(24) $\frac{dP}{dt} = kP(M-P)$

carrying capacity

$M=200$

$\boxed{A}$

(A) $\frac{dP}{dt} = .001P(200-P)$

(B) $\frac{dP}{dt} = .001P(100-P)$

(C) $\frac{dP}{dt} = .001P(200P-1)$

(D) $\frac{dP}{dt} = .001P(100P-1)$

(E) $\frac{dP}{dt} = .001P(100P+1)$

(25) $f(x) = \begin{cases} cx+d & x\le 2 \\ x^2-cx & x>2 \end{cases}$ $\quad f'(x) = \begin{cases} c & x\le 2 \\ 2x-c & x>2 \end{cases}$

$\lim_{x\to 2^-}(cx+d) = \lim_{x\to 2^+}(x^2-cx)$

$2c+d = 4-2c$

$4+d=4-4$

$4+d=0$

$d=-4$

$\lim_{x\to 2^-} c = \lim_{x\to 2^+}(2x-c)$

$c=4-c$

$2c=4$

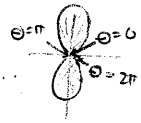
$c=2$

$c+d = 2+(-4) = -2$ $\boxed{B}$

(26) $\sin^2\theta = 0$

$\sin\theta = 0$

$\theta = 0, \pi, 2\pi, \dots$



$A = \frac{1}{2}\int_0^{2\pi}(\sin^2\theta)^2\,d\theta$

$= \frac{1}{2}\int_0^{2\pi}(\sin^4\theta)\,d\theta$

or $2\left(\frac{1}{2}\int_0^{\pi}(\sin^4\theta)\,d\theta\right)$

$\boxed{D}$ $\int_0^{\pi}(\sin^4\theta)\,d\theta$

(27) $\frac{dy}{dx} = y^2-1$

Depends only on y, so, not B, C, or D.

when $y=\pm 1$, $\frac{dy}{dx}=0$,

So, $\boxed{A}$

(28) $y=x^2-x$ $\qquad$ speed $=|v(t)| = 2\sqrt{10}$

$\frac{dy}{dt} = ?$

$|v(t)| = \sqrt{\left(\frac{dy}{dt}\right)^2+\left(\frac{dy}{dt}\right)^2}$

$2\sqrt{10} = \sqrt{\left(\frac{dx}{dt}\right)^2+\left((2x-1)\frac{dx}{dt}\right)^2}$

$2\sqrt{10} = \sqrt{\left(\frac{dy}{dt}\right)^2}\sqrt{1+(2x-1)^2}$

$2\sqrt{10} = \frac{dx}{dt}\sqrt{1+(2x-1)^2}$

$\frac{2\sqrt{10}}{\sqrt{1+(2\cdot 2-1)^2}} = \frac{dx}{dt}\Big|_{x=2}$

$\frac{2\sqrt{10}}{\sqrt{10}} = \frac{dx}{dt}\Big|_{x=2}$

$\frac{dx}{dt}\Big|_{x=2} = 2$

$\frac{dy}{dt} = 2x\frac{dx}{dt} - \frac{dx}{dt}$

$= (2x-1)\frac{dx}{dt}$

$\frac{dy}{dt}\Big|_{x=2} = (2(2)-1)(2)$

$= 3(2)$

$= 6$ $\boxed{D}$