2012

Answer Key for AP Calculus BC Practice Exam, Section I

Multiple-Choice Questions	
Question #	Key
• 1	E
2	A
• 3	B
4	A
5	C
• 6	C
• 7	A
• 8	C
9	D
• 10	E
• 11	A
• 12	C
13	C
14	E
• 15	A
16	C
17	C
• 18	A
• 19	C
20	C
• 21	E
22	D
• 23	A
24	E
25	B
26	B
27	C
28	D

•AB questions

$$\text{AB subscore} = \frac{\text{AB pts}}{13/28}$$

2012 BC
NonCalculator

(1) $y = (\sin x)^3$

$\frac{dy}{dx} = 3(\sin x)^2 \cdot \cos x$

$\boxed{E}$

(2) particle rest → velocity = 0

$\frac{dx}{dt} = 3t^2 - 6t$ $\quad \frac{dy}{dt} = 12 - 6t$

$0 = 3t(t-2)$ $\quad 0 = 12 - 6t$

$t = 0, t = 2$ $\quad 6t = 12$

$t = 2$

rest when $t = 2$

$x(2) = 2^3 - 3(2)^2 = 8 - 12 = -4$

$y(2) = 12(2) - 3(2)^2 = 24 - 12 = 12$

$\boxed{A}$ $(-4, 12)$

(3) $\int_0^4 f(x)\,dx$

$= \int_0^2 f(x)\,dx + \int_2^4 f(x)\,dx$

$= \frac{1}{2}(1+5)(2) + 2(-3)$

$= 6 - 6$

$= 0$

$\boxed{B}$

(4) $L = \int_1^2 \sqrt{1 + \left(\frac{1}{x}\right)^2}\,dx$

$\boxed{A}$

(5) $f(x) = \sum_{n=0}^{\infty} \left(-\frac{x}{4}\right)^n$

$f(3) = \sum_{n=0}^{\infty} \left(-\frac{3}{4}\right)^n$

geo series

$a_1 = 1, \; r = -\frac{3}{4}$

$\frac{1}{1 - -3/4} = \frac{1}{7/4} = \frac{4}{7}$

$\boxed{C}$

(6) $u = x^2 - 3$

$\frac{du}{dx} = 2x \rightarrow \frac{du}{2x} = dx$

$u(-1) = (-1)^2 - 3$

$= -2$

$u(4) = 4^2 - 3$

$= 13$

$\int_{-2}^{13} x(u)^5 \frac{du}{2x}$

$= \frac{1}{2}\int_{-2}^{13} u^5\,du$ $\boxed{C}$

(7) $\arcsin x = \ln y$

$\frac{1}{\sqrt{1-x^2}} = \frac{1}{y}\frac{dy}{dx}$

$\boxed{A}$ $\frac{y}{\sqrt{1-x^2}} = \frac{dy}{dx}$

(8) oil intake = initial + $\int_0^{15} R(t)\,dt$

right RAM $= 5.6(3) + 5.9(5) + 6.2(3)$

$= 16.8 + 29.5 + 18.6$

$= 64.9$

$$\begin{array}{r} 16.8 \\ 29.5 \\ 18.6 \\ \hline 64.9 \end{array}$$

$50 + 64.9$

$\boxed{C}$ $= 114.9$

(9) I. Ratio

$\lim_{n\to\infty} \frac{8^{n+1}}{(n+1)!} \cdot \frac{n!}{8^n}$

$= \lim_{n\to\infty} \frac{8}{n+1}$

$= 0$

converges $\forall x$ ✓

II. Ratio

$\lim_{n\to\infty} \frac{(n+1)!}{(n+1)^{100}} \cdot \frac{n^{100}}{n!}$

$= \lim_{n\to\infty} \frac{(n+1)n^{100}}{(n+1)^{100}}$ deg N > deg D so, ∞

$= \infty$

diverges

III. compares to $\frac{n}{n^3} = \frac{1}{n^2}$

$\sum \frac{1}{n^2}$ converges, p-series

so $\sum \frac{n+1}{n(n+2)(n+3)}$ also converges ✓

$\boxed{D}$ I and III

(10) $\int_1^4 t^{-3/2}\,dt$

$= -2t^{-1/2} \Big|_1^4$

$= -2\left(4^{-1/2} - 1^{-1/2}\right)$

$= -2\left(\frac{1}{2} - 1\right)$

$= -2\left(-\frac{1}{2}\right)$

$\boxed{E}$ $= 1$

(11) $f(x) = \sqrt{|x-2|} = \begin{cases} \sqrt{x-2} & \text{for } x \geq 2 \\ \sqrt{-(x-2)} & \text{for } x < 2 \end{cases}$

$f'(x) = \begin{cases} \frac{1}{2}(x-2)^{-1/2} = \frac{1}{2\sqrt{x-2}} & \text{for } x \geq 2 \\ \frac{1}{2}(-(x-2))^{-1/2}(-1) = -\frac{1}{2\sqrt{-(x-2)}} & \text{for } x < 2 \end{cases}$

$\left.\begin{array}{l} \lim_{x\to 2^-} f(x) = 0, \; f(2) = 0 \\ \lim_{x\to 2^+} f(x) = 0 \end{array}\right\} \therefore$, f cont @ $x = 2$

$\lim_{x\to 2} f'(x) =$ DNE, ∴ f not diff'able @ $x = 2$

$\boxed{A}$

(12) $\frac{dy}{dx} = x^2 + y$

$\frac{dy}{dx}\Big|_{(-1,-1)} = (-1)^2 + -1 = 0$

so, $(-1,-1)$ is crit pt

$\frac{dy}{dx}\Big|_{(1,-5)} = 1^2 + -5 = -4$

$(1,-5)$ not a crit pt.

$\frac{d^2y}{dx^2} = 2x + \frac{dy}{dx}$

$= 2x + x^2 + y$

$\frac{d^2y}{dx^2}\Big|_{(-1,-1)} = 2(-1) + (-1)^2 + -1$ 2nd derivative test

$= -2 + 1 - 1$

$= -2$

$< 0 \rightarrow \therefore$, $(-1,-1)$ is a Local max.

$\boxed{C}$

(13) geometric

$\sum_{n=1}^{\infty} \left(\frac{(x-4)^2}{3}\right)^n$

$|r| < 1$

$\left|\frac{(x-4)^2}{3}\right| < 1$

$|(x-4)^2| < 3$

$|x-4| < \sqrt{3}$ radius of conv.

$\boxed{C}$ $\sqrt{3}$

(14) $\frac{dP}{dt} = kP(M-P)$

$\frac{dy}{dt} = ky(M-y)$ $\boxed{E}$

(15) $h(6) = \int_0^6 f(t)\,dt$ area below x-axis

$\boxed{h(6) < 0}$

$h'(x) = f(x)$ $\quad h''(x) = f'(x)$

$h'(6) = f(6)$ $\quad h''(6) = f'(6)$

$h'(6) = 0$ $\quad h''(6) > 0$ f inc @ $x = 6$

$\boxed{A}$ $h(6) < h'(6) < h''(6)$

(16)

	dy/dx	$dy/dx\,\Delta x$	$\frac{dy}{dx}(\Delta x) + y$
$(1, 3)$	-2	$-2(.5) = -1$	$-1 + 3 = 2$
$(1.5, 2)$	$-.5$	$-.5(.5) = -.25$	$-.25 + 2 = 1.75$
$(2, 1.75)$			

→ $1\frac{3}{4} = \frac{7}{4}$ $\boxed{C}$

(17) $\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \ldots$

$\frac{\sin x}{x} = \frac{x}{x} - \frac{x^3}{3!}\cdot\frac{1}{x} + \frac{x^5}{5!}\cdot\frac{1}{x} - \ldots$

$\boxed{C}$

$= 1 - \frac{x^2}{3!} + \frac{x^4}{5!} - \ldots$

(18) $f(-5) = f(2) + \int_2^{-5} f'(t)\,dt$

$= 1 - \int_{-5}^{2} f'(t)\,dt$

$= 1 - \left[\text{area}_{\blacktriangle} + \text{area}_{\text{semicircle}}\right]$

$= 1 - \left[\frac{1}{2}(3)(2) + - \frac{\pi(2)^2}{2}\right]$

$= 1 - [\,3 - 2\pi\,]$

$\boxed{A}$ $= -2 + 2\pi$

(19) $f(x) = \frac{x}{x+2}$

$f'(x) = \frac{(x+2)(1) - x(1)}{(x+2)^2}$

$= \frac{x+2-x}{(x+2)^2}$

$\frac{1}{2} = \frac{2}{(x+2)^2}$

$(x+2)^2 = 4$

$x+2 = \pm 2$

$x = 0,\ x = -4$

$f(0) = \frac{0}{0+2} = 0 \quad f(-4) = \frac{-4}{-4+2} = \frac{-4}{-2} = 2$

$\boxed{C}$

(20) $\int_0^1 \frac{5x+8}{x^2+3x+2}\,dx$

$\frac{5x+8}{(x+2)(x+1)} = \frac{A}{x+2} + \frac{B}{x+1}$

$5x+8 = Ax + A + Bx + 2B$

$5 = A + B \qquad 8 = A + 2B$

$\hookrightarrow -5 = -A - B$

$A = 2 \qquad 3 = B$

$\int_0^1 \left(\frac{2}{x+2} + \frac{3}{x+1}\right) dx$

$= \left(2\ln|x+2| + 3\ln|x+1|\right)\Big|_0^1$

$= 2\ln 3 + 3\ln 2 - (2\ln 2 + 3\ln 1)$

$= 2\ln 3 + 3\ln 2 - 2\ln 2 + 0$

$= 2\ln 3 + \ln 2$

$= \ln 9 + \ln 2$

$= \ln(9 \cdot 2) = \ln 18$ $\boxed{C}$

(21) H.A $\rightarrow \lim_{x\to\infty} f(x)$

$\lim_{x\to\infty} \frac{20x^2 - x}{1+4x^2}$ deg N = deg D look @ Coefficients

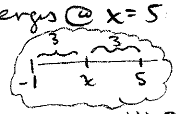
$= \frac{20}{4}$

$y = 5$ $\boxed{E}$

(22) $\sum_{n=0}^{\infty} a_n (x-3)^n$ converges @ $x = 5$,

$|x-3| < 2$

$1 < x < 5$



$\boxed{D}$ series converges @ $x = 2$ b/c 2 is on $(1,5)$

(23) linear growth $\rightarrow \frac{dP}{dt} = \text{constant}$

b/c lines have constant slope

$\boxed{A}$ $\frac{dP}{dt} = 200$

(24) $\int f(x)\sin x\,dx = -f(x)\cos x + \boxed{\int 4x^3 \cos x\,dx}$

$u = f(x) \qquad \int dv = \int \sin x\,dx$

$du = f'(x)dx \qquad v = -\cos x$

$uv - \int v\,du$

$f(x)\cdot -\cos x - \int -\cos x \cdot f'(x)\,dx$

$-f(x)\cos x + \boxed{\int f'(x)\cos x\,dx}$

so, $f'(x) = 4x^3$

$\int f'(x) = \int 4x^3\,dx$

$f(x) = x^4 + C$

$\boxed{E}$

(25) $\int_1^{\infty} x e^{-x^2}\,dx$

$= \lim_{a\to\infty} \int_1^a x e^{-x^2}\,dx$ $\qquad u = -x^2 \qquad u(a) = -a^2$

$du = -2x\,dx \qquad u(1) = -1$

$-\frac{1}{2}du = x\,dx$

$= \lim_{a\to\infty} \int_{-1}^{-a^2} -\frac{1}{2} e^u\,du$

$= \lim_{a\to\infty} -\frac{1}{2} e^u \Big|_{-1}^{-a^2}$

$= \lim_{a\to\infty} -\frac{1}{2}\left(e^{-a^2} - e^{-1}\right)$

$= -\frac{1}{2}(0 - e^{-1})$

$= \frac{1}{2}e^{-1} = \frac{1}{2e}$ $\boxed{B}$

(26) $\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta}$ $\qquad x = r\cos\theta = (1+2\sin\theta)(\cos\theta)$

$y = r\sin\theta = (1+2\sin\theta)(\sin\theta)$

$= \frac{\sin\theta(2\cos\theta) + (1+2\sin\theta)(\cos\theta)}{\cos\theta(2\cos\theta) + (1+2\sin\theta)(-\sin\theta)}$

$\frac{dy}{dx}\Big|_{\theta=0} = \frac{\sin 0(2\cos 0) + (1+2\sin 0)(\cos 0)}{\cos 0(2\cos 0) + (1+2\sin 0)(-\sin 0)}$

$= \frac{0+1}{1(2)+0} = \frac{1}{2}$ $\boxed{B}$

(27) $\sum_{n=1}^{\infty} \frac{1}{n^{2p}}$ converges when $2p > 1$

$p > \frac{1}{2}$

$\sum \left(\frac{p}{2}\right)^n$ converges when $\left|\frac{p}{2}\right| < 1$

$|p| < 2$

$-2 < p < 2$

$\therefore \frac{1}{2} < p < 2$

$\boxed{C}$

(28) $\lim_{x\to 1} \frac{\int_1^x g(t)\,dt}{g(x) - 6} = \frac{\int_1^1 g(t)\,dt}{g(1)-6} = \frac{0}{6-6} = \frac{0}{0}$ L'Hopital

$\lim_{x\to 1} \frac{g(x)}{g'(x)} = \frac{g(1)}{g'(1)} = \frac{6}{3} = 2$ $\boxed{D}$