2013

Answer Key for AP Calculus BC Practice Exam, Section I

- Question 1: C
- Question 2: C
- Question 3: D
- Question 4: E
- Question 5: A
- Question 6: D
- Question 7: C
- Question 8: D
- Question 9: D
- Question 10: D
- Question 11: D
- Question 12: E
- Question 13: D
- Question 14: B
- Question 15: B
- Question 16: A
- Question 17: B
- Question 18: A
- Question 19: C
- Question 20: E
- Question 21: A
- Question 22: D
- Question 23: C
- Question 24: A
- Question 25: C
- Question 26: D
- Question 27: A
- Question 28: A

• AB questions

AB subscore = $\frac{\text{AB pts}}{15/28}$

2013 | BC

(1) $f'(x) = \dfrac{(x+3)(2x+3) - (x^2+3x+2)(1)}{(x+3)^2}$

$= \dfrac{2x^2+9x+9-x^2-3x-2}{(x+3)^2}$

$= \dfrac{x^2+6x+7}{(x+3)^2}$

$\boxed{C}$

(2) $\int 5x(\sqrt{x} - x^2)\,dx$

$= \int (5x^{3/2} - 5x^3)\,dx$

$= 5(\frac{2}{5}x^{5/2}) - \frac{5}{4}x^4 + C$

$= 2x^{5/2} - \frac{5}{4}x^4 + C$

$\boxed{C}$

(3) $\sum_{n=1}^{\infty} \dfrac{(-3)^n \cdot -3^1}{5^n}$

$= \sum_{n=1}^{\infty} -3\left(\frac{-3}{5}\right)^n$

geo series

$S_\infty = \dfrac{a_1}{1-r} = \dfrac{9/5}{1-{-3/5}} = \dfrac{9/5}{8/5} = \dfrac{9}{8}$ $\boxed{D}$

(4) $y - y_1 = m(x - x_1)$

$y+3 = \frac{11}{3}(x-1)$

$\boxed{E}$

$x^2 - 3xy = 10$

$2x - 3(y \cdot 1 + x\frac{dy}{dx}) = 0$

$2(1) - 3(-3 + 1\frac{dy}{dx}) = 0$

$2 + 9 - 3\frac{dy}{dx} = 0$

$11 - 3\frac{dy}{dx} = 0$

$11 = 3\frac{dy}{dx}$

$\frac{11}{3} = \frac{dy}{dx}$

(5) $y = \frac{1}{2}x^{4/5} - \frac{3}{x^5}$

$y = \frac{1}{2}x^{4/5} - 3x^{-5}$

$\frac{dy}{dx} = \frac{1}{2}(\frac{4}{5}x^{-1/5}) + 15x^{-6}$

$\frac{dy}{dx} = \frac{2}{5}x^{-1/5} + 15x^{-6}$

$\boxed{A} = \dfrac{2}{5x^{1/5}} + \dfrac{15}{x^6}$

(6) $\int_0^3 |f(x)|\,dx$

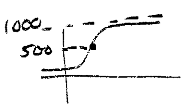
$= \int_0^1 f(x)\,dx + \int_1^2 f(x)\,dx - \int_2^3 f(x)\,dx$

$= \frac{1}{2}(1+2)(1) + \frac{1}{2}(1)(1) + \frac{1}{2}(1)(1)$

$= \frac{3}{2} + \frac{1}{2} + \frac{1}{2}$

$\boxed{D}$ $= \frac{5}{2}$

(7) $\frac{dy}{dt} = 0.2y(1000-y)$



inc @ dec rate

slopes positive but less steep

(concave down)

$\boxed{C}$ $500 < y < 1000$

(8) $L = \int_0^1 \sqrt{(x'(t))^2 + (y'(t))^2}\,dt$

$L = \int_0^1 \sqrt{(3)^2 + (2t)^2}\,dt$

$L = \int_0^1 \sqrt{9 + 4t^2}\,dt$

$\boxed{D}$

(9)

	Δx	$\frac{dy}{dx}$	$\frac{dy}{dx}\Delta x$	$y + \frac{dy}{dx}\Delta x$
(1, 0)	.5	2	1	0 + 1 = 1
(1.5, 1)	.5	4	2	1 + 2 = 3
(2, 3)				

$f(2) = 3$ $\boxed{D}$

(10) $\int_0^k \dfrac{x}{x^2+4}\,dx$

$u = x^2+4$

$\frac{du}{dx} = 2x$

$\frac{du}{2x} = dx$

$u(0) = 4$

$u(k) = k^2+4$

$\int_4^{k^2+4} \frac{x}{u} \cdot \frac{du}{2x}$

$\frac{1}{2}\int_4^{k^2+4} \frac{1}{u}\,du$

$\frac{1}{2} \ln u \Big|_4^{k^2+4}$

$\frac{1}{2}\left[\ln(k^2+4) - \ln 4\right] = \frac{1}{2}\ln 4$

$\ln(k^2+4) - \ln 4 = \ln 4$

$\ln(k^2+4) = 2\ln 4$

$\ln(k^2+4) = \ln 16$

$k^2 + 4 = 16$

$k^2 = 12$

$\boxed{D}$ $k = \sqrt{12}$

(11) $f(x) = 2 + \frac{1}{4}(x-4) - \frac{1}{64}(x-4)^2 + \frac{1}{512}(x-4)^3$

$f'(x) = \frac{1}{4} - \frac{1}{32}(x-4) + \frac{3}{512}(x-4)^2$

$f''(x) = -\frac{1}{32} + \frac{6}{512}(x-4)$

$f'''(x) = \frac{6}{512}$

$f'''(4) = \dfrac{\cancel{2} \cdot 3}{\cancel{2} \cdot 256}$

$\boxed{D}$ $= \frac{3}{256}$

(12) $\lim_{x\to\infty} f(x) = 0$

I. $\lim_{x\to\infty} \dfrac{\ln x}{x^{99}} = \dfrac{\infty}{\infty}$

$\lim_{x\to\infty} \dfrac{1/x}{99x^{98}} = \lim_{x\to\infty} \dfrac{1}{99x^{99}} = \dfrac{1}{\infty} = 0$ ✓

II. $\lim_{x\to\infty} \dfrac{e^x}{\ln x} = \dfrac{\infty}{\infty}$

$\lim_{x\to\infty} \dfrac{e^x}{1/x} = \lim_{x\to\infty} xe^x = \infty \cdot \infty = \infty$ ✗

III. $\lim_{x\to\infty} \dfrac{x^{99}}{e^x} = \dfrac{\infty}{\infty}$

$\lim_{x\to\infty} \dfrac{99x^{98}}{e^x} = \dfrac{\infty}{\infty}$

$\therefore \lim_{x\to\infty} \dfrac{99 \cdot 98 \cdot \ldots \cdot 2 \cdot 1}{e^x} = \dfrac{\#}{\infty} = 0$ ✓

I and III $\boxed{E}$

(13) $f(5) = 20 + \int_0^5 f'(x)\,dx$

$\approx 20 + 15$

≈ 35 $\boxed{D}$

$\int_0^5 f'(x)\,dx > 2(5) = 10$

and $\int_0^5 f'(x)\,dx < 4(5) = 20$

(14) $\lim_{x\to\infty} \dfrac{\ln(bx+1)}{\ln(ax^2+3)} = \dfrac{\infty}{\infty}$

$\lim_{x\to\infty} \dfrac{\frac{1}{bx+1} \cdot b}{\frac{1}{ax^2+3} \cdot 2ax} = \lim_{x\to\infty} \dfrac{\frac{b}{bx+1}}{\frac{2ax}{ax^2+3}} = \lim_{x\to\infty} \dfrac{b}{bx+1} \cdot \dfrac{ax^2+3}{2ax}$

$= \lim_{x\to\infty} \dfrac{abx^2 + 3b}{2abx^2 + 2ax}$

$= \dfrac{ab}{2ab} = \dfrac{1}{2}$ $\boxed{B}$

(15) *Ratio Test*

$\lim_{n\to\infty} \dfrac{\frac{(-1)^{n+1}}{n+1}(x+3/2)^{n+1}}{\frac{(-1)^n}{n}(x+3/2)^n}$

$= \lim_{n\to\infty} (-1)(x+3/2)^1\left(\frac{n+1}{n}\right)$

$= -1(x+3/2)$

$|-1(x+3/2)| < 1$

$|x+3/2| < 1$

$-1 < x + 3/2 < 1$

$-5/2 < x < -1/2$

endpts

$x = -5/2$ $\sum_{n=1}^{\infty} \dfrac{(-1)^n(-1)^n}{n}$

$\sum_{n=1}^{\infty} \dfrac{1}{n}$

diverge $p = 1$

$x = -1/2$ $\sum_{n=1}^{\infty} \dfrac{(-1)^n(1)^n}{n}$

$= \sum_{n=1}^{\infty} \dfrac{(-1)^n}{n}$

AST

converge

so, $-5/2 < x \le -1/2$ $\boxed{B}$

16

$$\int \frac{1}{100P-P^2}\,dP$$

$$= \int \frac{1}{P(100-P)}\,dP \qquad \frac{1}{P(100-P)} = \frac{A}{P} + \frac{B}{100-P}$$

$$= \frac{1}{100}\left[\int \frac{1}{P}\,dP + \int \frac{1}{100-P}\,dP\right] \qquad 1 = A(100) - AP + BP$$

$$= \frac{1}{100}\left(\ln P + -\ln(100-P) + c\right) \qquad 100A = 1,\ A = \frac{1}{100} \qquad -A+B=0,\ B=A$$

$\boxed{A}$

17

$$\lim_{h\to 0} \frac{\arcsin(a+h) - \arcsin(a)}{h} = 2$$

$f'(x)$ where $f(x) = \arcsin x$

$f'(x) = \frac{1}{\sqrt{1-x^2}}$

$2 = \frac{1}{\sqrt{1-x^2}}$

$2\sqrt{1-x^2} = 1$

$\sqrt{1-x^2} = 1/2$

$1-x^2 = 1/4$

$-x^2 = -3/4$

$x^2 = 3/4$

$x = \sqrt{3}/2$

$\boxed{B}$

18

$h(x) = f(g(x))$

$h'(x) = f'(g(x))\cdot g'(x)$

$h'(1) = f'(g(1))\cdot g'(1)$

$h'(1) = f'(2(1)+1)\cdot 2$

$= f'(3)\cdot 2$

$= -2\cdot 2$

$= -4$ $\boxed{A}$

$f'(3) = \frac{f(4)-f(1)}{4-1} = \frac{-2-4}{3} = -2$

19

$\frac{1}{1-x} = 1 + x + x^2 + x^3 + \dots$

$\frac{d}{dx}\left(\frac{1}{1-x}\right) = \frac{d}{dx}(1 + x + x^2 + x^3 + \dots)$

$-(1-x)^{-2}(-1) = 1 + 2x + 3x^2 + \dots$

$\frac{1}{(1-x)^2} = 1 + 2x + 3x^2 + \dots$

$\boxed{C}$

20

$y - y_1 = m(x - x_1)$

$g(5) = \int_0^5 f(t)\,dt$

$= 2(-2)\frac{1}{2}$

$= -2$

$g'(x) = \frac{d}{dx}\int_0^x f(t)\,dt$

$g'(x) = f(x)$

$g'(5) = f(5)$

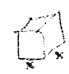
$= -1$ $\boxed{E}$

$y + 2 = -1(x-2)$

21

$\frac{dV}{dt} = k\,SA$

$SA = 6x^2$



$\frac{dV}{dt} = k(6x^2)$

$\boxed{A}$

22

$2^2 - 2(1) + 1^2 = 3$

$4 - 2 + 1 = 3$

$3 = 3$ ✓

$2x - (y\cdot 1 + x\frac{dy}{dx}) + 2y\frac{dy}{dx} = 0$

$2x - y - x\frac{dy}{dx} + 2y\frac{dy}{dx} = 0$

$\frac{dy}{dx}(-x+2y) = -2x+y$

$\frac{dy}{dx} = \frac{-2x+y}{-x+2y}$

$\left.\frac{dy}{dx}\right|_{(2,1)} = \frac{-4+1}{-2+2} = \frac{-3}{0}$

$\boxed{D}$

23

$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots$

$e^{3x^2} = 1 + (3x^2) + \frac{(3x^2)^2}{2!} + \frac{(3x^2)^3}{3!} + \dots$

$\frac{e^{3x^2}}{2} = \frac{1}{2} + \frac{3}{2}x^2 + \frac{9x^4}{2\cdot 2!} + \underbrace{\frac{27x^6}{2\cdot 3!}} + \dots$

$\frac{27}{2\cdot 3\cdot 2\cdot 1}$

$= \frac{3\cdot 3\cdot \cancel{3}}{2\cdot \cancel{3}\cdot 2} = \frac{9}{4}$ $\boxed{C}$

24 abs min value → lowest y-value

$g'(x) = 12x^2 + 6x - 6$

$0 = 6(2x^2 + x - 1)$

$0 = 6(2x-1)(x+1)$

$x = 1/2,\ x = -1$

g': $+\ -\ +$; $(-2)\ -1\ (0)\ 1/2\ (1)$

rel. min @ $x = 1/2$

$g(1/2) = 4(1/2)^3 + 3(1/2)^2 - 6(1/2) + 1$

$= 4(1/8) + 3/4 - 3 + 1$

$= 1/2 + 3/4 - 2$

$= 5/4 - 8/4 = -3/4$

check endpts

$g(-2) = 4(-2)^3 + 3(-2)^2 - 6(-2) + 1$

$= 4(-8) + 3(4) + 12 + 1$

$= -32 + 12 + 12 + 1$

$= -7 \longrightarrow$ lowest $\boxed{A}$

$g(1) = 4(1)^3 + 3(1)^2 - 6(1) + 1$

$= 4 + 3 - 6 + 1$

$= 2$

25

$dy = e^{y+x}\,dx$

$dy = e^y\cdot e^x\,dx$

$\int \frac{1}{e^y}\,dy = \int e^x\,dx$

$\int e^{-y}\,dy = e^x + C$

$-e^{-y} = e^x + C \rightarrow -e^{-y} = e^x - 5$

$-e^{-(\ln 4)} = e^0 + C$

$\ln e^{-y} = \ln(-e^x + 5)$

$-e^{\ln 4} = 1 + C$

$-y = \ln(-e^x + 5)$

$-4 = 1 + C$

$y = -\ln(-e^x + 5)$

$-5 = C$

$\boxed{C}$

26

I. $\sum_{n=1}^{\infty} \frac{|\sin n|}{n^2}$ ✓ $\quad |\sin n| < 1$, so use $\sum_{n=1}^{\infty} \frac{1}{n^2}$ → converge by p-test $p > 1$

II $\sum_{n=1}^{\infty} e^{-n} = \sum_{n=1}^{\infty} (e^{-1})^n$ ✓

geo series where $e^{-1} < 1$, $\frac{1}{e} < 1$

so converges

× III. $\sum_{n=1}^{\infty} \frac{n+2}{n^2+n}$ compares to $\sum_{n=1}^{\infty} \frac{n}{n^2}$, $\sum_{n=1}^{\infty} \frac{1}{n}$ diverges p-series $p=1$

I and II $\boxed{D}$

27

$\int_1^{\infty} f(t)\,dt$

$= \lim_{x\to\infty} \int_1^x f(t)\,dt$

$= \lim_{x\to\infty}\left(\frac{20x}{\sqrt{4x^2+21}} - 4\right)$

$= \lim_{x\to\infty}\left(\frac{20x}{\sqrt{4x^2+21}}\right) - \lim_{x\to\infty}(4)$

$=$ exp N = exp D $- 4$

$\frac{20x}{\sqrt{4x^2}} \approx \frac{20x}{\sqrt{4}\,x}$

$= \frac{20}{2} - 4$

$= 10 - 4$

$= 6$ $\boxed{A}$

28

$$\frac{d^2y}{dx^2} = \frac{\frac{d}{dt}\left(\frac{dy}{dx}\right)}{\frac{dx}{dt}}$$

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{\frac{1}{t}}{2t} = \frac{1}{t}\cdot\frac{1}{2t} = \frac{1}{2t^2} = \frac{1}{2}t^{-2}$$

$$\frac{d^2y}{dx^2} = \frac{\frac{d}{dt}\left(\frac{1}{2}t^{-2}\right)}{dx/dt} = \frac{-t^{-3}}{2t} = -\frac{1}{t^3}\cdot\frac{1}{2t} = -\frac{1}{2t^4}$$

$\boxed{A}$