2014

Answer Key for AP Calculus BC Practice Exam, Section I

Question 1: C

Question 2: B

Question 3: D

Question 4: E

Question 5: C

Question 6: E

Question 7: A

Question 8: B

Question 9: A

Question 10: B

Question 11: E

Question 12: C

Question 13: E

Question 14: E

Question 15: B

Question 16: A

Question 17: E

Question 18: B

Question 19: B

Question 20: B

Question 21: D

Question 22: B

Question 23: D

Question 24: C

Question 25: B

Question 26: A

Question 27: D

Question 28: B

$\frac{\text{AB pts}}{15/28}$ = AB subscore pts

2014 | BC | Noncalculator

① $\int \frac{x^3+5}{x^2}\,dx$

$= \int\left(\frac{x^3}{x^2}+\frac{5}{x^2}\right)dx$

$= \int(x + 5x^{-2})\,dx$

$= \frac{1}{2}x^2 - 5x^{-1} + C$

$\boxed{C}$

② $y' = \frac{1}{2x}\cdot 2$

$y' = \frac{1}{x}$

$y'(4) = \frac{1}{4}$

$\boxed{B}$

③ $\lim_{x\to 0}\frac{x^2}{1-\cos x} = \frac{0}{1-1} = \frac{0}{0}$, L'Hopital

$\lim_{x\to 0}\frac{2x}{\sin x} = \frac{0}{0}$

$\lim_{x\to 0}\frac{2}{\cos x} = \frac{2}{1} = 2$

$\boxed{D}$

④ $\int\frac{1}{(x-5)(x-2)}\,dx$

$\frac{1}{(x-5)(x-2)} = \frac{A}{x-5} + \frac{B}{x-2}$

$1 = A(x-2) + B(x-5)$

$1 = Ax - 2A + Bx - 5B$

$A + B = 0 \qquad 1 = -2A - 5B$

$B = -A \qquad 1 = -2A - 5(-A)$

$B = -\frac{1}{3} \qquad 1 = 3A$

$\frac{1}{3} = A$

$\int\left(\frac{1/3}{x-5} - \frac{1/3}{x-2}\right)dx$

$= \frac{1}{3}\int\left(\frac{1}{x-5} - \frac{1}{x-2}\right)dx$

$= \frac{1}{3}\left(\ln|x-5| - \ln|x-2|\right) + C$

$= \frac{1}{3}\ln\left|\frac{x-5}{x-2}\right| + C \quad \boxed{E}$

⑤ I. $\lim_{x\to 2^-} f(x) = f(2)$

$-1 = -1$ ✓

II. $\lim_{x\to 6^-} f(x) = \lim_{x\to 6^+} f(x)$

$-1 = -1$ ✓

III $\lim_{x\to 6} f(x) = f(6)$

$-1 \neq -3$ ✗

I and II only $\boxed{C}$

⑥ $\sum_{n=1}^{\infty} a_k \approx (-1)^{n+1}$

$\approx -1+1-1+1$

series diverges

$\boxed{E}$

⑦ f cont → $f(3) = \lim_{x\to 3} f(x)$

need,

$\lim_{x\to 3^-} f(x) = \lim_{x\to 3^+} f(x)$

$\lim_{x\to 3^-}(x^2+2) = \lim_{x\to 3^+}(6x+k)$

$11 = 18 + k$

$-7 = k$

$\boxed{A}$

⑧ $\int_0^1 x\sqrt{1+8x^2}\,dx$

$u = 1+8x^2 \qquad u(1) = 9$

$du = 16x\,dx \qquad u(0) = 1$

$\frac{1}{16}du = x\,dx$

$\frac{1}{16}\int_1^9 u^{1/2}\,du$

$= \frac{1}{16}\left(\frac{2}{3}u^{3/2}\right)\Big|_1^9$

$= \frac{1}{24}\left[9^{3/2} - 1^{3/2}\right]$

$= \frac{1}{24}[27-1]$

$= \frac{26}{24} = \frac{13}{12} \quad \boxed{B}$

⑨ rel. max → f' changes pos to neg.

$f' = x(x-3)^2(x+1)$

$0 = x(x-3)^2(x+1)$

$x=0,\ x=3,\ x=-1$

f': $+$ | $-$ | $+$ | $+$

-3, -1, $-\frac{1}{2}$, 0, 1, 3, 4

rel. max @ $x = -1$

$\boxed{A}$

⑩ $\sum_{n=1}^{\infty}\frac{(-2)^n}{e^{n+1}} = \sum_{n=1}^{\infty}\frac{(-2)^n}{e^n\cdot e} = \frac{1}{e}\sum_{n=1}^{\infty}\left(\frac{-2}{e}\right)^n$

geometric

$r = \frac{-2}{e}$

$a = \frac{1}{e}\left(\frac{-2}{e}\right) = \frac{-2}{e^2}$

$= \frac{-2/e^2}{1 - -2/e}$

$= \frac{-2/e^2}{\frac{e+2}{e}}$

$= \frac{-2}{e^2}\cdot\frac{e}{e+2}$

$= \frac{-2}{e(e+2)} = \frac{-2}{e^2+2e} \quad \boxed{B}$

⑪ $f(x) = \begin{cases} 2x+5 & x < -1 \\ -x^2+6 & x \geq -1 \end{cases}$

f needs to be cont.

$\left.\begin{array}{l}\lim_{x\to -1^-} f(x) = 2(-1)+5 = 3 \\ \lim_{x\to -1^+} f(x) = -(-1)^2+6 = 5\end{array}\right\}\neq$,

$\therefore$ f not cont,

so f not diff'able @ $x = -1$

$\boxed{E}$

⑫ 9^x

$\Delta x = \frac{2-0}{4}$

$\Delta x = \frac{1}{2}$

x	0	½	1	3/2	2
f(x)		3	9	27	81

right

$\int_0^2 f(x)\,dx = \frac{1}{2}(81+27+9+3)$

$= \frac{1}{2}(90+30)$

$= \frac{1}{2}(120)$

$= 60 \quad \boxed{C}$

⑬ WALL

x | x

y

$P = 18$ m

$A_{max} = ?$

$P = 2x + y$

$18 = 2x + y$

$18 - 2x = y$

$A = xy$

$A = x(18-2x)$

$A = 18x - 2x^2$

$A' = 18 - 4x$

$0 = 18 - 4x$

$4x = 18$

$x = 9/2$

$A = 9/2(18 - 2(9/2))$

$= 9/2(18-9)$

$= 9/2(9) = 81/2\ m^2 \quad \boxed{E}$

⑭ $P(x) = 3 - 3x^2 + 6x^4$

$\frac{f^{(4)}(0)}{4!} = 6$

$f^{(4)}(0) = 6(4)!$

$= 6\cdot 4\cdot 3\cdot 2$

$= 24\cdot 3\cdot 2$

$= 72\cdot 2$

$= 144$

$\boxed{E}$

⑮ $\ln x - \ln y = y - 4$

$y = 4$
$x = 4$

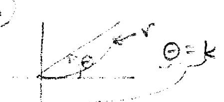
$\frac{1}{x} - \frac{1}{y}\frac{dy}{dx} = \frac{dy}{dx}$

$\frac{1}{4} - \frac{1}{4}\frac{dy}{dx} = \frac{dy}{dx}$

$\frac{1}{4} = \frac{dy}{dx} + \frac{1}{4}\frac{dy}{dx}$

$\frac{1}{4} = \frac{5}{4}\frac{dy}{dx}$

$\boxed{B} \qquad \frac{1}{5} = \frac{dy}{dx}$

⑯ 

$\theta = k$

$\frac{1}{2}\int_0^{\theta} R^2\,d\theta$

$r = \theta$

$\frac{1}{2}\int_0^k \theta^2\,d\theta$

$\frac{1}{2}\left(\frac{1}{3}\theta^3\right)\Big|_0^k$

$\boxed{A} \quad \frac{1}{6}k^3$

⑰ $e^x = 1 + x + \frac{x^2}{2} + \frac{x^3}{3!} + \ldots$

$e^{3x} = 1 + 3x + \frac{(3x)^2}{2} + \frac{(3x)^3}{3!} + \ldots$

$= 1 + 3x + \frac{9x^2}{2} + \frac{27x^3}{3!} + \ldots$

$\boxed{E}$

(18) $\int_1^\infty \frac{x^2}{(x^3+2)^2}\,dx$

$u = x^3+2$
$du = 3x^2\,dx$
$\frac{1}{3}du = x^2\,dx$

$\frac{1}{3}\int u^{-2}\,du$

$-\frac{1}{3}u^{-1} = -\frac{1}{3u}$

$-\frac{1}{3}\left(\frac{1}{x^3+2}\right)\Big|_1^\infty$

$-\frac{1}{3}(0) - -\frac{1}{3}\left(\frac{1}{1^3+2}\right)$

$\frac{1}{3}\cdot\frac{1}{3}$

$\boxed{B}$ $\frac{1}{9}$

(19) pt inf → y'' change signs

$y = 3x^5 + 10x^4$

$y' = 15x^4 + 40x^3$

$y'' = 60x^3 + 120x^2$

$0 = 60x^2(x+2)$
$x = 0,\ x = -2$

y'': − | + | +
(−3) −2 (−1) 0 (1)

inf pt @ $x = -2$

$\boxed{B}$

(20) $g(x) = f(x^2-1)$

$g'(x) = f'(x^2-1)\cdot 2x$

$g'(2) = f'(3)\cdot 2(2)$

$= 4f'(3)$

$= 4\left(\frac{5}{16}\right)$

$\boxed{B}$ $= \frac{5}{4}$

$f'(x) = \frac{(x-2)^3(x^2-4)}{16}$

$f'(3) = \frac{(3-2)^3(3^2-4)}{16}$

$= \frac{(1)(5)}{16}$

$= \frac{5}{16}$

(21) length of curve $= \int_1^{?} \sqrt{1+(f'(x))^2}\,dx$

$= 2\left(\sqrt{1+2^2}\right) + 2\left(\sqrt{1+1^2}\right) + 2\left(\sqrt{1+0^2}\right)$

$\boxed{D}$ $= 2\sqrt{5} + 2\sqrt{2} + 2$

(22) $\sum_{n=1}^{\infty} \frac{(x-3)^n}{n\cdot 2^n}$

$\lim_{n\to\infty} \frac{(x-3)^{n+1}}{(n+1)2^{n+1}} \cdot \frac{n\cdot 2^n}{(x-3)^n}$

$\lim_{n\to\infty} \frac{(x-3)n}{2(n+1)}$

$= \frac{x-3}{2}$

$\left|\frac{x-3}{2}\right| < 1$

$|x-3| < 2$

$-2 < x-3 < 2$

$1 < x < 5$

check endpts

$x=1$ $\sum_{n=1}^{\infty} \frac{(-2)^n}{n\cdot 2^n} = \sum_{n=1}^{\infty} (-1)^n \frac{1}{n}$

converges, AST

$x=5$ $\sum_{n=1}^{\infty} \frac{2^n}{n\cdot 2^n} = \sum_{n=1}^{\infty} \frac{1}{n}$

diverges, p-series $p=1$

$\boxed{B}$ $1 \le x < 5$

(23) $\frac{dy}{dx} = xy^2$ $y(2) = 1$

$\int \frac{1}{y^2}\,dy = \int x\,dx$

$\int y^{-2}\,dy = \frac{1}{2}x^2 + C$

$-y^{-1} = \frac{1}{2}x^2 + C$

$-(1)^{-1} = \frac{1}{2}(2)^2 + C$

$-1 = 2 + C$

$-3 = C$

$-\frac{1}{y} = \frac{1}{2}x^2 - 3$

$\frac{1}{y} = -\frac{1}{2}x^2 + 3$

$\frac{1}{y} = \frac{-x^2+6}{2}$

$\boxed{D}$ $y = \frac{2}{-x^2+6}$

(24) I. $1 + (-1) + 1 + \dots (-1)^{n-1} + \dots$
diverges, geo w/ $r = -1$

II. $1 + \frac{1}{3} + \frac{1}{5} + \dots \frac{1}{2n-1} + \dots$

LCT $a_n = \frac{1}{2n-1}$ $b_n = \frac{1}{n}$

$\lim_{n\to\infty} \frac{\frac{1}{2n-1}}{\frac{1}{n}} = \lim_{n\to\infty} \frac{1}{2n-1}\cdot\frac{n}{1} = \frac{1}{2}$, so

Since $\sum \frac{1}{n}$ diverges by p-series, the $\sum \frac{1}{2n-1}$ also diverges

III. $1 + \frac{1}{3} + \frac{1}{3^2} + \dots \frac{1}{3^{n-1}} + \dots$

geometric w/ $r = \frac{1}{3}$

converges

$\boxed{C}$ III only

(25) $\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta}$

$x = r\cos\theta$ $y = r\sin\theta$
$x = (\cos\theta)^2$ $y = \cos\theta\sin\theta$

$dy/dx = \frac{\sin\theta(-\sin\theta) + \cos\theta(\cos\theta)}{2\cos\theta(-\sin\theta)}$

$= \frac{-(\sin\theta)^2 + (\cos\theta)^2}{-2\sin\theta\cos\theta}$

$dy/dx\Big|_{\theta=\pi/6} = \frac{-(\sin\frac{\pi}{6})^2 + (\cos\frac{\pi}{6})^2}{-2\sin\frac{\pi}{6}\cos\frac{\pi}{6}}$

$= \frac{-(\frac{1}{2})^2 + (\frac{\sqrt{3}}{2})^2}{-2(\frac{1}{2})(\frac{\sqrt{3}}{2})}$

$= \frac{-\frac{1}{4} + \frac{3}{4}}{-\sqrt{3}/2}$

$= \frac{\frac{1}{2}}{-\frac{\sqrt{3}}{2}} = \frac{1}{2}\cdot\frac{2}{\sqrt{3}} = -\frac{1}{\sqrt{3}}$

$\boxed{B}$

(26) $\frac{d}{dx}\int_1^{\sqrt{x}} \frac{1}{1+t^2}\,dt$ 2nd FTC

$= \frac{1}{1+(\sqrt{x})^2}\cdot\frac{1}{2}x^{-1/2}$

$= \frac{1}{1+x}\cdot\frac{1}{2\sqrt{x}}$

$= \frac{1}{2\sqrt{x}(1+x)}$ $\boxed{A}$

(27) $f(x) = (\sin x)^2$

$f'(x) = 2\sin x\cos x$

$f''(x) = 2(\cos x\cdot\cos x + \sin x(-\sin x))$

$= 2(\cos^2 x - \sin^2 x)$

$f''(0) = 2(\cos^2 0 - \sin^2 0)$

$= 2$

$(\sin x)^2 = \dots + \underbrace{\frac{2}{2!}}_{1}(x-0)^2 + \dots$

$\boxed{D}$

(28) $h(x) = x^5 + 3x - 2$

$h'(x) = 5x^4 + 3$

$h(1) = 2$ $h^{-1}(2) = 1$

$h'(1) = 8$ $(h^{-1})'(2) = \frac{1}{8}$

reciprocal

$\boxed{B}$