2015

Answer Key for AP Calculus BC Practice Exam, Section I

- Question 1: E
- Question 2: B
- Question 3: E
- Question 4: A
- Question 5: D
- Question 6: B
- Question 7: D
- Question 8: A
- Question 9: C
- Question 10: E
- Question 11: B
- Question 12: C
- Question 13: C
- Question 14: D
- Question 15: C
- Question 16: B
- Question 17: B
- Question 18: B
- Question 19: C
- Question 20: C
- Question 21: B
- Question 22: D
- Question 23: D
- Question 24: D
- Question 25: B
- Question 26: A
- Question 27: A
- Question 28: B

$\frac{\text{AB pts}}{14/28}$ = AB subscore pts.

2015 BC
NonCalculator

① $\frac{dy}{dx} = \frac{(x^2+1)(2x) - (x^2-2)(2x)}{(x^2+1)^2}$

$\left.\frac{dy}{dx}\right|_{x=1} = \frac{(1^2+1)(2\cdot1) - (1^2-2)(2\cdot1)}{(1^2+1)^2}$

$= \frac{4-(-2)}{4}$

$= \frac{6}{4} = \frac{3}{2}$

$\boxed{E}$

② $y^2 - 2x^2y = 8$

$2y\frac{dy}{dx} - 2(y\cdot 2x + x^2\frac{dy}{dx}) = 0$

$2y\frac{dy}{dx} - 4xy - 2x^2\frac{dy}{dx} = 0$

$\frac{dy}{dx}(2y - 2x^2) = 4xy$

$\frac{dy}{dx} = \frac{4xy}{2y-2x^2}$

$\frac{dy}{dx} = \frac{\cancel{2}(2xy)}{\cancel{2}(y-x^2)}$

$\frac{dy}{dx} = \frac{2xy}{y-x^2}$

$\boxed{B}$

③ $\int x^2(x^3+5)^6\,dx$

$u = x^3+5$
$du = 3x^2\,dx$
$\frac{1}{3}du = x^2\,dx$

$\frac{1}{3}\int u^6\,du$

$\frac{1}{3}(\frac{1}{7}u^7) + c$

$\frac{1}{21}(x^3+5)^7 + c$

$\boxed{E}$

④ $\int_0^{50} f(x)\,dx$

$= 25(4) + 5(6) + 20(8)$
$= 100 + 30 + 160$
$= 100 + 190$
$= 290$

$\boxed{A}$

⑤ $L = \int_a^b \sqrt{1+(f'(x))^2}\,dx$

$y = \sqrt{x}$
$y' = \frac{1}{2}x^{-1/2}$

$= \int_1^4 \sqrt{1+(\frac{1}{2}x^{-1/2})^2}\,dx$

$= \int_1^4 \sqrt{1+\frac{1}{4}x^{-1}}\,dx$

$= \int_1^4 \sqrt{1+\frac{1}{4x}}\,dx$

$\boxed{D}$

⑥ $\int \frac{6}{(x+8)(x+2)}\,dx$

$\frac{6}{(x+8)(x+2)} = \frac{A}{x+8} + \frac{B}{x+2}$

$6 = Ax + 2A + Bx + 8B$

$0 = A + B \qquad 6 = 2A + 8B$
$B = -A \qquad 6 = 2A + 8(-A)$
$\qquad 6 = -6A$
$B = 1 \qquad -1 = A$

$\int \frac{-1}{x+8}dx + \int \frac{1}{x+2}dx$

$= -\ln|x+8| + \ln|x+2| + C$

$= \ln|x+2| - \ln|x+8| + C$

$= \ln\left|\frac{x+2}{x+8}\right| + C$

$\boxed{B}$

⑦ derivative of $f(g(x))$

$= f'(g(x))\cdot g'(x)$

$f(x) = x^2 - 4$
$f'(x) = 2x$
$f'(g(x)) = 2g(x)$

$= 2g(x)\cdot g'(x)$

$\boxed{D}$

⑧ moving? velocity

$\langle x'(t), y'(t)\rangle = \langle -2, 2t\rangle$

neg for x, so moves left

pos for y, so moves up

y-position
$t^2 - 3 = -2$
$t^2 = 5$
$t = \sqrt{5}$

$\boxed{A}$

⑨ $\Delta x = \frac{0-1}{2} = -\frac{1}{2}$

	dy/dx	$\frac{dy}{dx}(\Delta x)$	$\frac{dy}{dx}(\Delta x) + y_1$
$(1,2)$	3	$3(-\frac{1}{2}) = -\frac{3}{2}$	$-3/2 + 2 = \frac{1}{2}$
$(\frac{1}{2},\frac{1}{2})$	$\frac{1}{2}$	$\frac{1}{2}(-\frac{1}{2}) = -\frac{1}{4}$	$-\frac{1}{4} + \frac{1}{2} = \frac{1}{4}$
$(0, \frac{1}{4})$			

$\boxed{C}$

⑩
A $\sum \frac{3n}{n+2}$ compares to $\sum 3$ diverges

B $\sum \frac{3n}{n^2+2}$ compares to $\sum \frac{3}{n}$ diverges p-series

C $\sum \frac{3n}{n^2+2n}$ compares to $\sum \frac{3}{n}$ see above

D. $\sum \frac{3n^2}{n^3+2n}$ compares to $\sum \frac{3}{n}$

E. $\sum \frac{3n^2}{n^4+2n}$ compares to $\sum \frac{3}{n^2}$ converges, p-series

$\boxed{E}$

⑪ $\int (2^t + e^\pi)\,dt$

$= 2^t\cdot\frac{1}{\ln 2} + e^\pi(t) + C$

$\boxed{B}$

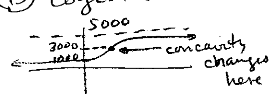
⑫ $\lim_{x\to0}\frac{e^x-1}{x} = \frac{e^0-1}{0} = \frac{0}{0}$

L'Hopital

$\lim_{x\to0}\frac{e^x}{1} = \frac{1}{1} = 1$

$\boxed{C}$

⑬ Logistic



$\frac{dP}{dt} = 5P(\frac{1}{5000})(5000-P)$

$= \frac{1}{1000}P(5000-P)$

carrying capacity

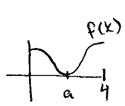
I. $\lim_{t\to\infty} P(t) = 5000$ ✓

II $\frac{dP}{dt} > 0$ for $t > 0$ ✓ (see graph)

III. $\frac{d^2P}{dt^2} > 0$ for $t > 0$ ✗ (see graph)

I and II only $\boxed{C}$

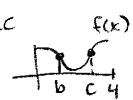
⑭ f dec → $f' < 0$ on $(0,a)$



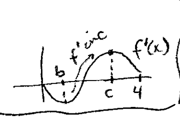
f inc → $f' > 0$ on $(a,4)$

So, C or D

f conc. up → f' inc on (b,c)



f conc. down → f' dec on $(0,b)\cup(c,4)$



so $\boxed{D}$

⑮ Area $= \frac{1}{2}\int_0^{\pi/4}\left(\frac{2}{\cos\theta+\sin\theta}\right)^2 d\theta$

$= \frac{1}{2}\int_0^{\pi/4}\frac{4}{(\cos\theta+\sin\theta)^2}\,d\theta$

$= 2\int_0^{\pi/4}\frac{1}{(\cos\theta+\sin\theta)^2}\,d\theta$

$\boxed{C}$

⑯ $1 + \frac{2^1}{1!} + \frac{2^2}{2!} + \frac{2^3}{3!} + \dots + \frac{2^n}{n!} + \dots$

$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots$

$e^2 = 1 + 2 + \frac{2^2}{2!} + \frac{2^3}{3!} + \dots$

$\boxed{B}$

⑰ $\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{4t^3}{2t} = 2t^2$

$\frac{d^2y}{dx^2} = \frac{\frac{d}{dt}(\frac{dy}{dx})}{dx/dt} = \frac{4t}{2t} = 2$

$\boxed{B}$

⑱ $y(6) = y(0) + \int_0^6 -10e^{-t/2}\,dt$

$u = -t/2$
$du = -\frac{1}{2}dt$
$-2\,du = dt$

$= 20 + \left(20e^{-t/2}\right)\Big|_0^6$

$= 20 + 20(e^{-6/2} - e^0)$

$= 20 + 20e^{-3} - 20$

$= 20e^{-3}$

$\boxed{B}$

⑲ $f(x) = f(0) + f'(0)x + \dots \frac{f'''(0)}{3!}(x)^3 + \dots$

need $f'''(0)$

$f''(x) = \sqrt{1+3x}$

$f'''(x) = \frac{1}{2}(1+3x)^{-1/2}(3)$

$f'''(0) = \frac{3}{2}$

coefficent $= \frac{3/2}{3!}$

$= \frac{3}{2}\cdot\frac{1}{3\cdot2}$

$= \frac{1}{4}$

$\boxed{C}$

(20) $\sum_{n=0}^{\infty} \frac{(x-4)^n}{n \cdot 3^{n+1}}$

Ratio

$\lim_{n\to\infty} \frac{(x-4)^{n+1}}{(n+1)3^{n+2}} \cdot \frac{n \cdot 3^{n+1}}{(x-4)^n}$

$\lim_{n\to\infty} \frac{(x-4)n}{3(n+1)}$

$\left|\frac{x-4}{3}\right| < 1$

$|x-4| < 3$ ← radius of convergence: 3

$\boxed{C}$

(21) $\int_1^{\infty} \frac{1}{x^p}\,dx$ diverges when $p=1, p<1$

$\int_1^0 \frac{1}{x^p}\,dx$ diverges when $p=1, p>1$

$\boxed{B}$

(22) H.A. → $\lim_{x\to\infty} f(x)$

$\lim_{x\to\infty} \frac{2x}{\sqrt{x^2-1}} = \frac{2}{\pm 1} = \pm 2$

$\boxed{D}$ $y=2, y=-2$

(23) $F'(x) = \frac{d}{dx}\int_4^{x^2} \sqrt{t}\,dt$

$= \sqrt{x^2} \cdot 2x$

$= x \cdot 2x$

$\boxed{D}$ $= 2x^2$

(24) $\frac{dy}{dx} = -2xy$

$\int \frac{1}{y}\,dy = \int 2x\,dx$

$\ln|y| = -x^2 + C$ → $y(1)=4$

$\ln|4| = -1^2 + C$

$\ln 4 = -1 + C$

$1 + \ln 4 = C$

$e^{\ln|y|} = e^{-x^2+1+\ln 4}$

→ $|y| = e^{-x^2} \cdot e^1 \cdot e^{\ln 4}$

$y = \pm e^{-x^2} \cdot e^1 \cdot 4$

$y = 4e^{-x^2+1}$ $\boxed{D}$

keep pos b/c $y(1)=4$ posi

(25) def. of derivative

$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$

$f'(\frac{\pi}{3}) = \lim_{h\to 0} \frac{\sin(\frac{\pi}{3}+h) - \sin(\frac{\pi}{3})}{h}$

$f(x) = \sin x$ @ $x = \pi/3$

$f'(x) = \cos x$

$f'(\pi/3) = \cos \pi/3$

$= \frac{1}{2}$ $\boxed{B}$

(26) g dec → $g' < 0$

$g'(x) = \frac{d}{dx}\int_{-1}^{x} \frac{t^3-t^2-6t}{\sqrt{t^2+7}}\,dt$

$0 = \frac{x^3-x^2-6x}{\sqrt{x^2+7}}$

$0 = x(x^2-x-6)$

$0 = x(x-3)(x+2)$

$x=0, x=3, x=-2$

g': $-$, $+$, $-$, $+$; -3, -2, -1, 0, 1, 3, 4

$\boxed{A}$ g dec on $(-\infty,-2)\cup(0,3)$

(27) $f(x) = \sin x + 2x + 1$

$f'(x) = \cos x + 2$

$f(x) = 1$

$\sin x + 2x + 1 = 1$

$\sin x + 2x = 0$

when $x=0$

$f(0) = 1$ $g(1) = 0$

$f'(0) = 3$ $g'(1) = \frac{1}{3}$

reciprocal

$\boxed{A}$

$f'(0) = \cos 0 + 2$

$= 1 + 2$

$= 3$

(28) $|f(1) - P_3(1)| \le \frac{\max|f^{(4)}(c)|}{4!} x^4$

$\le \frac{\frac{4}{5}}{4!}(1)^4$

$\le \frac{4}{5} \cdot \frac{1}{4!}$

$|f^{(4)}(x)| \le \frac{4}{4+1}$

$\le \frac{4}{5}$

$\boxed{B}$