DATE: ____________

1. Use the definition of the derivative to find $f'(x)$ of $f(x) = x^2 + 2x + 1$ at (-3,4).

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{(x+h)^2 + 2(x+h) + 1 - (x^2 + 2x + 1)}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{x^2} + 2xh + h^2 + \cancel{2x} + 2h + \cancel{1} - \cancel{x^2} - \cancel{2x} - \cancel{1}}{h}$$

$$= \lim_{h \to 0} \frac{2xh + h^2 + 2h}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{h}(2x + h + 2)}{\cancel{h}}$$

$$\boxed{f'(x) = 2x + 2}$$

2. Use the alternate form of the derivative to find $f'(x)$ of $f(x) = x^2 - 3x$ at $x = 1$.

$$f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

$$f(1) = 1^2 - 3(1) = -2$$

$$f'(1) = \lim_{x \to 1} \frac{x^2 - 3x - (-2)}{x - 1}$$

$$= \lim_{x \to 1} \frac{x^2 - 3x + 2}{x - 1}$$

$$= \lim_{x \to 1} \frac{(x-2)\cancel{(x-1)}}{\cancel{x-1}}$$

$$\boxed{f'(1) = -1}$$

3. If $y = \dfrac{1}{x+3}$, then find $\dfrac{dy}{dx}$.

$$\frac{dy}{dx} = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{\frac{1}{x+h+3} - \frac{1}{x+3}}{h}$$

$$= \lim_{h \to 0} \frac{\frac{(x+3)}{(x+3)}\frac{1}{x+h+3} - \frac{1}{x+3}\frac{(x+h+3)}{(x+h+3)}}{h}$$

$$= \lim_{h \to 0} \frac{\frac{x+3}{(x+3)(x+h+3)} - \frac{x+h+3}{(x+3)(x+h+3)}}{h}$$

$$= \lim_{h \to 0} \frac{\frac{\cancel{x}+\cancel{3} - \cancel{x} - h - \cancel{3}}{(x+3)(x+h+3)}}{h}$$

$$= \lim_{h \to 0} \frac{-h}{(x+3)(x+h+3)} \div h$$

$$\frac{dy}{dx} = \lim_{h \to 0} \frac{-\cancel{h}}{(x+3)(x+h+3)} \cdot \frac{1}{\cancel{h}}$$

$$= \lim_{h \to 0} \frac{-1}{(x+3)(x+h+3)}$$

$$\boxed{\frac{dy}{dx} = \frac{-1}{(x+3)^2}}$$

4. Find $\frac{d}{dx}(3x^2 - 5)$.

$$\frac{d}{dx}(3x^2-5) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$$

$$= \lim_{h\to 0} \frac{3(x+h)^2-5-(3x^2-5)}{h}$$

$$= \lim_{h\to 0} \frac{3(x^2+2xh+h^2)-5-3x^2+5}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{3x^2}+6xh+3h^2-\cancel{5}-\cancel{3x^2}+\cancel{5}}{h}$$

$$= \lim_{h\to 0} \frac{6xh+3h^2}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{h}(6x+3h)}{\cancel{h}}$$

$$\boxed{\frac{d}{dx}(3x^2-5) = 6x}$$

5. Find the equation of the line tangent to the graph of $f(x) = x^2 + 1$ at the point (1, 2)

$$y - y_1 = m(x - x_1)$$

$$\boxed{y - 2 = 2(x-1)}$$

alternate form

$$f'(1) = \lim_{x\to 1} \frac{f(x)-f(1)}{x-1}$$

$$= \lim_{x\to 1} \frac{x^2+1-(2)}{x-1}$$

$$= \lim_{x\to 1} \frac{x^2-1}{x-1}$$

$$= \lim_{x\to 1} \frac{\cancel{(x-1)}(x+1)}{\cancel{x-1}}$$

$$f'(1) = 2$$