Chain Rule: Station 1

Let $h(x) = f(g(x))$. Use the table below to answer the following questions.

x	1	2	3
$f(x)$	–2	8	1
$f'(x)$	3	2	4
$g(x)$	1	–3	2
$g'(x)$	4	1	–3

1. $h(3) = f(g(3))$
$= f(2)$
$= \boxed{8}$

2. $h'(x) = g'(x) \cdot f'(g(x))$

3. $h'(1) = g'(1) \cdot f'(g(1))$
$= 4 \cdot f'(1)$
$= 4 \cdot 3 = \boxed{12}$

4. Write the equation of the tangent line to $h(x)$ at $x = 3$.

$y - y_1 = m(x - x_1)$
$y - h(3) = h'(3)(x-3)$
$\boxed{y - 8 = -6(x-3)}$

$h'(3) = g'(3) \cdot f'(g(3))$
$= -3 \cdot f'(2)$
$= -3 \cdot 2$
$= -6$

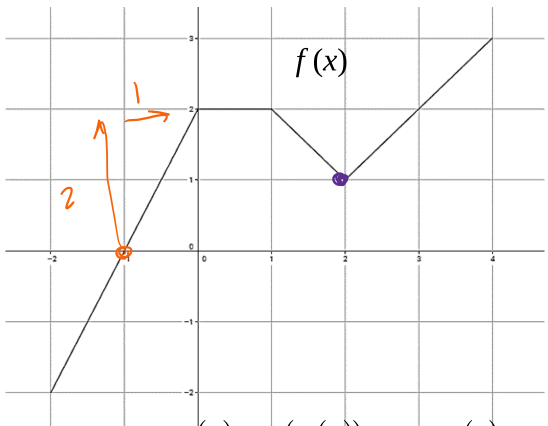
Chain Rule: Station 2

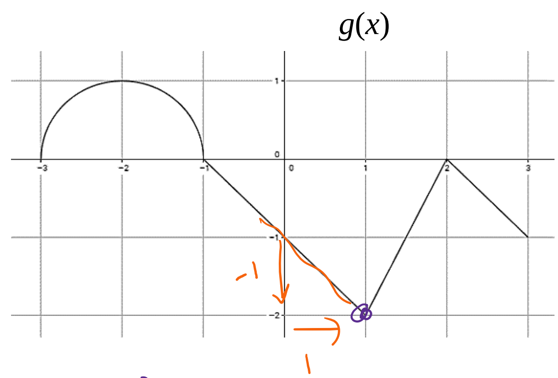
1. $f(x) = \sin(3x)$
$f'(x) = 3\cos(3x)$

2. $f(x) = x^2 \cos(2x+3)$
$f'(x) = \cos(2x+3) \cdot 2x + x^2 \cdot 2 \cdot -\sin(2x+3)$
$= 2x\cos(2x+3) - 2x^2 \sin(2x+3)$

3. $g(x) = \sin^2(3x^2 - 2x + 1) = (\sin(3x^2 - 2x + 1))^2$
$g'(x) = (6x-2)\cos(3x^2-2x+1) \cdot 2(\sin(3x^2-2x+1))^1$
$= (12x-4)\cos(3x^2-2x+1)\sin(3x^2-6x+1)$
$= 4(3x-1)\cos(3x^2-2x+1)\sin(3x^2-6x+1)$

Chain Rule: Station 3





1. If $h(x)=g(f(x))$, find $h(2)$

$h(2) = g(f(2))$
$= g(1)$
$= \boxed{-2}$

2. If $h(x)=g(f(x))$, find $h'(-1)$

$h'(x) = f'(x)\, g'(f(x))$

$h'(-1) = f'(-1)\, g'(f(-1))$
$= 2\, g'(0)$
$= 2(-1) = \boxed{-2}$

3. If $k(x)=f(x^3)$, find $k'(-1)$

$k'(x) = 3x^2 \cdot f'(x^3)$
$k'(-1) = 3(-1)^2\, f'((-1)^3)$

$k'(-1) = 3(1)\, f'(-1)$
$= 3 \cdot 2$
$= \boxed{6}$

4. Find the equation of the tangent line to $k(x)$ at $x=-1$.

$y - y_1 = m(x - x_1)$
$y - k(-1) = k'(-1)(x - -1)$
$y - 0 = 6(x+1)$
$\boxed{y = 6(x+1)}$

$k(-1) = f((-1)^3)$
$= f(-1)$
$= 0$

Chain Rule: Station 4

Given that $s(x)=f(x^3)$, use the table below to answer the following questions.

x	-8	-2	-1	1
$f(x)$	3	1	-1	2
$f'(x)$	4	-3	2	1

1. $s(-1) = f((-1)^3) = f(-1) = \boxed{-1}$

2. $s'(x) = 3x^2 \cdot f'(x^3)$

3. $s'(1) = 3(1)^2 \cdot f'(1)$
$= 3 \cdot 1 = \boxed{3}$

4. Write the equation of the tangent line to $s(x)$ when $x = -2$

$y - y_1 = m(x - x_1)$
$y - s(-2) = s'(-2)(x - -2)$
$\boxed{y - 3 = 48(x+2)}$

$s(-2) = f((-2)^3)$
$= f(-8)$
$= 3$

$s'(-2) = 3(-2)^2 \cdot f'((-2)^3)$
$= 3(4)\, f'(-8)$
$= 12 \cdot 4$
$= 48$

Chain Rule: Station 5

1. $f(x)=\sqrt{2x} = \sqrt{2}\sqrt{x} = \sqrt{2}\, x^{1/2}$ $\quad f'(x) = \frac{1}{2}\cdot\sqrt{2}\, x^{-1/2} = \frac{\sqrt{2}}{2\sqrt{x}}$

$f'(2)= \frac{\sqrt{2}}{2\sqrt{2}} = \boxed{\frac{1}{2}}$

2. $f(x)=\left(x^2-2x-1\right)^{\frac{2}{3}}$ $\quad f'(x) = (2x-2)\cdot\frac{2}{3}(x^2-2x-1)^{-1/3}$

$= \frac{4x-4}{3(x^2-2x-1)^{1/3}} = \frac{4x-4}{3\sqrt[3]{x^2-2x-1}}$

$f'(0)= \frac{4(0)-4}{3\sqrt[3]{0^2-2(0)-1}}$

$= \boxed{\frac{4}{3}}$

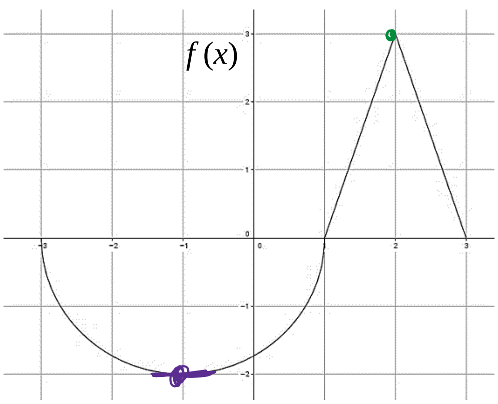
3. $f(x)=x\sqrt{2x-3} = x(2x-3)^{1/2}$

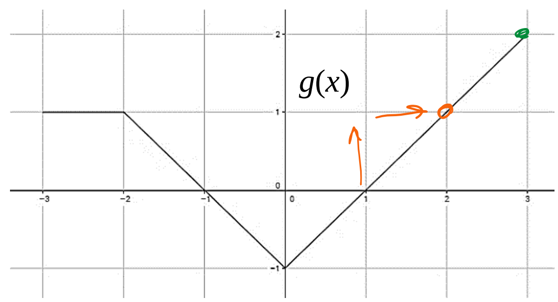
$f'(x)= (2x-3)^{1/2}\cdot 1 + x\cdot 2\cdot\frac{1}{2}(2x-3)^{-1/2}$

$= (2x-3)^{1/2} + x(2x-3)^{-1/2}$

$= (2x-3)^{1/2} + \frac{x}{(2x-3)^{1/2}} = \frac{2x-3}{(2x-3)^{1/2}} + \frac{x}{(2x-3)^{1/2}} = \frac{3x-3}{(2x-3)^{1/2}}$

Chain Rule: Station 6





1. If $h(x)=f(g(x))$, find $h(3)$

$h(3) = f(g(3))$

$= f(2) = \boxed{3}$

2. If $h(x)=f(g(x))$, find $h'(2)$.

$h'(x) = g'(x)\cdot f'(g(x))$

$h'(2) = g'(2)\, f'(g(2))$

$= 1\cdot f'(1)$

$=$ DNE b/c $f'(1)$ DNE b/c $\lim_{x\to 1^-} f'(x) \neq \lim_{x\to 1^+} f'(x)$

3. If $p(x)=g\left(x^2-x\right)$, find $p'(-1)$

$p'(x) = (2x-1)\, g'(x^2-x)$

$p'(-1) = (2(-1)-1)\, g'((-1)^2-(-1))$

$= -3\, g'(2)$

$= -3\cdot 1$

$= \boxed{-3}$

4. If $q(x)=\frac{f(x)}{(3x-1)^2}$, find $q'(-1)$

$q'(x) = \frac{(3x-1)^2 f'(x) - f(x)\cdot 3\cdot 2(3x-1)}{(3x-1)^4}$

$q'(-1) = \frac{(3(-1)-1)^2 f'(-1) - f(-1)\, 6(3(-1)-1)}{(3(-1)-1)^4}$

$= \frac{(-4)^2(0) - (-2)(6)(-4)}{(-4)^4}$

$= \frac{-48}{256}$

$= \boxed{\frac{-3}{16}}$