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Derivatives Practice

Understanding Derivatives

1) Given $\lim_{h\to 0}\frac{2(-1+h)^3+4-(2(-1)^3+4)}{h}$ as an expression for the derivative of $f(x)$ at $x=c$, identify the function $f(x)$ and the value of c.

definition of derivative

$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$

where $x=-1$

$f'(-1) = \lim_{h\to 0} \frac{f(-1+h)-f(-1)}{h}$

$f(-1) = 2(-1)^3+4$

so,

$f(x) = 2x^3+4$

$\boxed{\therefore,\ f(x) = 2x^3+4 \text{ and } c=-1}$

2) Given $\lim_{x\to 4}\frac{\ln(3x-2)-\ln 10}{x-4}$ as an expression for the derivative of $g(x)$ at $x=c$, identify the function $g(x)$ and the value of c.

alternate form of the derivative

$f'(a) = \lim_{x\to a} \frac{f(x)-f(a)}{x-a}$

where $a=4$

$f'(4) = \lim_{x\to 4} \frac{f(x)-f(4)}{x-4}$

$f(x) = \ln(3x-2)$

and $f(4) = \ln(3\cdot 4-2)$
$= \ln 10$

$\boxed{\therefore,\ f(x) = \ln(3x-2) \text{ and } c=4}$

3) An equation for the line tangent to the graph of the function f at $x=-2$ is $y+4=\frac{1}{5}(x+2)$. What is $f'(-2)$?

means slope of tangent line @ $x=-2$

$y-y_1 = m(x-x_1)$

slope of tangent line

$\boxed{f'(-2) = \frac{1}{5}}$

Derivatives Numerically

4) The table below gives select values for the differentiable function f, find the best estimate for $f'(12)$ that can be made from the given table.

approximation

slope of tangent $\approx$ slope of secant $\approx \frac{\Delta y}{\Delta x}$

x	6	7	10	13	15
$f(x)$	-1	5	2	-3	-6

$$f'(12) \approx \frac{f(13) - f(10)}{13 - 10}$$

$$\approx \frac{-3-2}{3} = \boxed{-2.667}$$

Derivatives using TI-Nspire

Using your graphing calculator (remember answers need to be with 3 decimal places), given:

5) $f(x) = x^2 e^{\cos x}$, find $f'(2)$.

$$\boxed{f'(2) = 0.239}$$

$$\frac{d}{dx}\left(x^2 \cdot e^{\cos(x)}\right)\Big|_{x=2} \qquad 0.239304$$

Derivatives Analytically (Algebraically)

6) Using the definition of the derivative, find the derivative of the function $f(x) = x^2 + 3x - 4$.

$$f'(x) = \lim_{h \to 0} \frac{(x+h)^2 + 3(x+h) - 4 - (x^2 + 3x - 4)}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{x^2} + 2xh + h^2 + \cancel{3x} + 3h \cancel{-4} \cancel{-x^2} \cancel{-3x} \cancel{+4}}{h}$$

$$= \lim_{h \to 0} \frac{2xh + h^2 + 3h}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{h}(2x + h + 3)}{\cancel{h}}$$

$$= \lim_{h \to 0} (2x + h + 3)$$

$$\boxed{f'(x) = 2x + 3}$$

7) Using the definition of the derivative, find the $g'(3)$ where $g(x) = \frac{1}{x}$.

$$g'(3) = \lim_{h\to 0} \frac{g(3+h) - g(3)}{h}$$

$$= \lim_{h\to 0} \frac{\frac{1}{3+h} - \frac{1}{3}}{h}$$

$$= \lim_{h\to 0} \frac{\frac{3}{3}\cdot\frac{1}{3+h} - \frac{1}{3}\cdot\frac{3+h}{3+h}}{h}$$

common denominators!

$$= \lim_{h\to 0} \frac{\frac{3-(3+h)}{3(3+h)}}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{3} - \cancel{3} - h}{3(3+h)} \cdot \frac{1}{h}$$

$$= \lim_{h\to 0} \frac{-\cancel{h}}{3(3+h)} \cdot \frac{1}{\cancel{h}}$$

$$= \lim_{h\to 0} \frac{-1}{3(3+h)}$$

$$= \frac{-1}{3(3)}$$

$$\boxed{g'(3) = -\frac{1}{9}}$$

8) Using the alternate form of the derivative, find slope of the tangent line to $h(x) = \sqrt{x}$ at $x = 4$.

$$h'(4) = \lim_{x\to 4} \frac{\sqrt{x} - \sqrt{4}}{x-4}$$

$$= \lim_{x\to 4} \frac{\sqrt{x} - 2}{x-4} \cdot \frac{\sqrt{x}+2}{\sqrt{x}+2}$$

multiply by the conjugate

$$= \lim_{x\to 4} \frac{\cancel{x-4}}{\cancel{(x-4)}(\sqrt{x}+2)}$$

$$= \lim_{x\to 4} \frac{1}{\sqrt{x}+2}$$

$$= \frac{1}{\sqrt{4}+2}$$

$$= \frac{1}{2+2}$$

$$h'(4) = \frac{1}{4}$$