AP Differentiability FRQs Solutions with Point Values

(a) $\lim_{h\to 0^-} \frac{f(h)-f(0)}{h} = \frac{2}{3}$

$\lim_{h\to 0^+} \frac{f(h)-f(0)}{h} < 0$

Since the one-sided limits do not agree, f is not differentiable at $x = 0$.

$$2 : \begin{cases} 1 : \text{sets up difference quotient at } x = 0 \\ 1 : \text{answer with justification} \end{cases}$$

Since g is continuous at $x = 3$, $2k = 3m + 2$.

$$g'(x) = \begin{cases} \frac{k}{2\sqrt{x+1}} & \text{for } 0 < x < 3 \\ m & \text{for } 3 < x < 5 \end{cases}$$

$\lim_{x\to 3^-} g'(x) = \frac{k}{4}$ and $\lim_{x\to 3^+} g'(x) = m$

Since these two limits exist and g is differentiable at $x = 3$, the two limits are equal. Thus $\frac{k}{4} = m$.

$8m = 3m + 2$; $m = \frac{2}{5}$ and $k = \frac{8}{5}$

$$3 : \begin{cases} 1 : 2k = 3m + 2 \\ 1 : \frac{k}{4} = m \\ 1 : \text{values for } k \text{ and } m \end{cases}$$