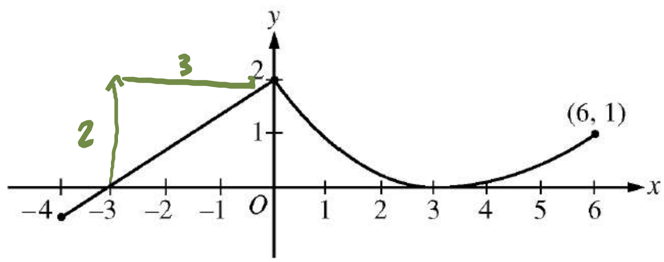
Date: _______________

## AP Differentiability FRQs




Graph of $f$

3. A continuous function $f$ is defined on the closed interval $-4 \le x \le 6$. The graph of $f$ consists of a line segment and a curve that is tangent to the $x$-axis at $x = 3$, as shown in the figure above. On the interval $0 < x < 6$, the function $f$ is twice differentiable, with $f''(x) > 0$.

   (a) Is $f$ differentiable at $x = 0$? Use the definition of the derivative with one-sided limits to justify your answer.

$\hookrightarrow \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$ where $x = 0$

left side

$$\lim_{h \to 0^-} \frac{f(0+h) - f(0)}{h} = \frac{3}{2}$$

right side

$$\lim_{h \to 0^+} \frac{f(0+h) - f(0)}{h} < 0$$

$\frac{3}{2} \neq 0$, $\therefore$ f is not diff'able @ $x = 0$

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Suppose the function $g$ is defined by

$$g(x) = \begin{cases} k\sqrt{x+1} & \text{for } 0 \le x \le 3 \\ mx + 2 & \text{for } 3 < x \le 5, \end{cases} \longrightarrow g'(x) = \begin{cases} \frac{1}{2}k(x+1)^{-1/2} & 0 \le x \le 3 \\ m & 3 < x \le 5 \end{cases}$$

where $k$ and $m$ are constants. If $g$ is differentiable at $x = 3$, what are the values of $k$ and $m$?

$\hookrightarrow$ g is cont @ $x = 3$

$$\lim_{x \to 3^-} k\sqrt{x+1} = \lim_{x \to 3^+} (mx + 2)$$

$$k\sqrt{3+1} = m(3) + 2$$

$$2k = 3m + 2$$

$$\lim_{x \to 3^-} g'(x) = \lim_{x \to 3^+} g'(x)$$

$$\lim_{x \to 3^-} \left(\frac{1}{2}k(x+1)^{-1/2}\right) = \lim_{x \to 3^+} m$$

$$\frac{1}{2}k(3+1)^{-1/2} = m$$

$$\frac{1}{2\sqrt{4}}k = m$$

$$\frac{1}{4}k = m$$

$$k = 4m$$

$$2(4m) = 3m + 2$$

$$8m = 3m + 2$$

$$5m = 2$$

$$\boxed{m = \frac{2}{5}}$$

$$k = 4\left(\frac{2}{5}\right)$$

$$\boxed{k = \frac{8}{5}}$$