DATE: ________________

3.2 Differentiability

Stating a function is differentiable at a point means:

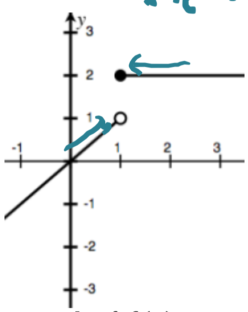
- able to differentiate
- able to get the derivative
- can find the slope of the tangent line
- can find the slope of the curve at a given point

When is a function NOT differentiable?

A function is NOT differentiable at $x = c$, when

① the function is NOT continuous @ $x = c$.
(jumps, holes, or vertical asymptotes)

"Jump" $\lim_{x \to c^-} f(x) \neq \lim_{x \to c^+} f(x)$



Graph of $f(x)$

$\lim_{x \to 1^-} f(x) \neq \lim_{x \to 1^+} f(x)$

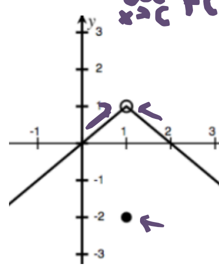
$1 \neq 2$

so $\lim_{x \to 1} f(x)$ DNE

$f(x)$ is not cont @ $x=1$

∴, $f(x)$ is not diff'able @ $x = 1$

"Hole" $f(c)$ DNE or $\lim_{x \to c} f(x) \neq f(c)$



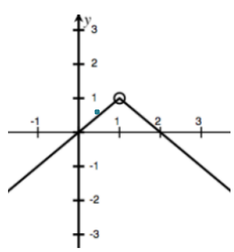
Graph of $g(x)$

$\lim_{x \to 1} g(x) \neq g(1)$

$1 \neq -2$

so $g(x)$ is not cont @ $x=1$

∴, $g'(x)$ @ $x=1$ DNE



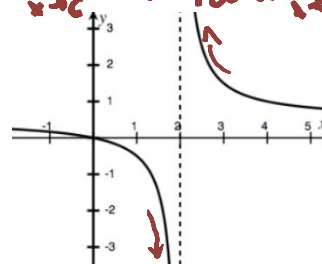
Graph of $h(x)$

$h(1)$ DNE

so $h(x)$ is not cont @ $x=1$

∴, $h'(1)$ DNE

"Vertical Asymptote" $\lim_{x \to c^-} f(x) = \pm\infty$ or $\lim_{x \to c^+} f(x) = \pm\infty$



Graph of $f(x)$

$\lim_{x \to 2^-} f(x) = -\infty$

(also, $\lim_{x \to 2^+} f(x) = \infty$)

so f is not cont @ $x = 2$

∴, f is not diff'able @ $x = 2$

② the function's derivative from the left of c is not equal to the function's derivative from the right of c. (sharp turn)

$g(x) = |2x+2| = \begin{cases} -(2x+2) & x \le -1 \\ 2x+2 & x > -1 \end{cases}$

$g'(x) = \begin{cases} -2 & x \le -1 \\ 2 & x > -1 \end{cases}$

$\lim_{x \to -1^-} g'(x) \neq \lim_{x \to -1^+} g'(x)$

$-2 \neq 2$

so g is not diff'able @ $x = -1$ (or $g'(-1)$ DNE)

③ the function's derivative is $\pm\infty$. (tangent line is vertical)

$h(x) = \sqrt[3]{x-3} + 2$

$h'(3) = \infty$ or $h'(3) = -\infty$

so $h'(3)$ DNE

$f(x) = \sqrt[3]{x} + 2$

$= x^{1/3} + 2$

$f'(x) = \frac{1}{3} x^{-2/3}$

$f'(0) = \frac{1}{3} \cdot \frac{1}{(\sqrt[3]{0})^2}$

$= \frac{1}{3} \cdot \frac{1}{0}$

$= \frac{1}{0}$ undefined

$f'(0)$ DNE