

## 3.2 Exponential &amp; Logistic Modeling

Target 3F: Model real world situations and use regressions with the use of functions

Review of Prior Concepts

The population ( $P$ ) of a city can be represented in an equation  $P = 3000e^{kt}$ , where  $t = 0$  represents the year 1900. In 1850, the population was 1100. Find the value of  $k$  and use this value of  $k$  to estimate the population in the year 2012.

$$P = 3000e^{kt}$$

$$1100 = 3000e^{k(-50)}$$

$$\frac{1100}{3000} = e^{-50k}$$

$$\ln\left(\frac{11}{30}\right) = \ln e^{-50k}$$

$$\ln\left(\frac{11}{30}\right) = -50k$$

$$\frac{\ln\left(\frac{11}{30}\right)}{-50} = k$$

$$.020 = k$$

Store in calculator

$$t = 0 \rightarrow 1900$$

$$t = -50 \rightarrow 1850$$

$$k = ?$$

then, when  $t = 112 \rightarrow 2012$ ,  
pop = ?

$$P = 3000e^{.020t}$$

$$P(112) = 3000e^{.020(112)}$$

$$= 28389.206$$

28389 people in 2012

## More Practice

## Population Modeling

<http://www.coolmath.com/algebra/17-exponentials-logarithms/06-population-exponential-growth-01>
<http://www.purplemath.com/modules/expoprob2.htm>
<https://www.youtube.com/watch?v=63udRYh04sY>


## SAT Connection

## Passport to Advanced Mathematics

1. Create a quadratic or exponential function or equation that models a context.

Example: A radioactive substance decays at an annual rate of 13 percent. If the initial amount of the substance is 325 grams, which of the following functions  $f$  models the remaining amount of the substance, in grams,  $t$  years later?

$$y_0 = 325$$

A)  $f(t) = 325(0.87)^t$

B)  $f(t) = 325(0.13)^t$

C)  $f(t) = 0.87(325)^t$

D)  $f(t) = 0.13(325)^t$

$$1 - .13 = .87$$

$$y = y_0(1+r)^t$$

$$y = 325(.87)^t$$

Solution

## Logistic Model

$$f(x) = \frac{c}{1 + a \cdot b^x}$$

- Limiting factor
- Limit to growth
- maximum sustainable population
- carrying capacity

Example 1:  
p.297 #24



$$f(x) = \frac{c}{1 + a \cdot b^x}$$

$$f(x) = \frac{60}{1 + a \cdot b^x}$$

$$12 = \frac{60}{1 + a \cdot b^0}$$

$$12 = \frac{60}{1 + a}$$

$$(1+a)12 = \frac{60(1+a)}{1+a}$$

$$12 + 12a = 60$$

$$12a = 48$$

$$a = 4$$

Limit to growth = 60

initial value = 12  
when  $x=0$ ,  $y=12$

$$f(x) = \frac{60}{1 + 4 \cdot b^x}$$

$$24 = \frac{60}{1 + 4 \cdot b^1}$$

$$24 = \frac{60}{1 + 4b}$$

$$(1+4b)24 = \frac{60}{1+4b} (1+4b)$$

$$24 + 96b = 60$$

$$96b = 36$$

$$b = \frac{3}{8}$$

pass thru  
(1,24)

$$f(x) = \frac{60}{1 + 4\left(\frac{3}{8}\right)^x}$$

Example 2:  
p.297 #26

$$f(x) = \frac{c}{1+a \cdot b^x} \quad \text{limit to growth} = 30$$

$$f(x) = \frac{30}{1+a \cdot b^x}$$

$$5 = \frac{30}{1+a \cdot b^0} \quad \text{initial value} = 5$$

when  $x=0, y=5$

$$5 = \frac{30}{1+a}$$

$$(1+a)5 = \frac{30}{1+a} (1+a)$$

$$5+5a = 30$$

$$5a = 25$$

$$a = 5$$

$$f(x) = \frac{30}{1+5 \cdot b^x} \quad \text{pass thru } (3, 15)$$

$$15 = \frac{30}{1+5 \cdot b^3}$$

$$15 = \frac{30}{1+5b^3}$$

$$(1+5b^3)15 = \frac{30}{1+5b^3} (1+5b^3)$$

$$15+75b^3 = 30$$

$$75b^3 = 15$$

$$b^3 = \frac{15}{75}$$

$$\sqrt[3]{b^3} = \sqrt[3]{\frac{1}{5}}$$

$$b = 0.585$$

$$f(x) = \frac{30}{1+5(0.585)^x}$$

Example 3:

a) initial population = ?

$$t = 0$$

$$P(0) = \frac{1001}{1+90e^{-2(0)}}$$

$$= \frac{1001}{1+90e^0}$$

... 😊  
 $e^0 = 1$

$$= \frac{1001}{1+90}$$

$$= \frac{1001}{91} = \boxed{11 \text{ deer}}$$

c) maximum # of deer  
(maximum sustainable population)

$$\boxed{1001 \text{ deer}}$$

b) # of deer = 600

$$600 = \frac{1001}{1+90e^{-2t}}$$

$$(1+90e^{-2t})600 = \frac{1001}{1+90e^{-2t}} (1+90e^{-2t})$$

$$600 + 54000e^{-2t} = 1001$$

$$54000e^{-2t} = 401$$

$$e^{-2t} = \frac{401}{54000}$$

$$\ln e^{-2t} = \ln \left( \frac{401}{54000} \right)$$

$$-2t = \ln \left( \frac{401}{54000} \right)$$

$$t = \frac{\ln \left( \frac{401}{54000} \right)}{-2}$$

$$t = 24.514$$

$$\boxed{24.514 \text{ years}}$$

**More Practice**

**Logistic Models**

<http://www.ck12.org/book/CK-12-Precalculus-Concepts/section/3.7/>

<https://www.youtube.com/watch?v=LyJrUtzKtwI>

<https://www.youtube.com/watch?v=OSMPeY354cU>

**Homework Assignment**

p.297 #23,28,45,47,50

**SAT Connection****Solution**

**Choice A is correct.** Each year, the amount of the radioactive substance is reduced by 13 percent from the prior year's amount; that is, each year, 87 percent of the previous year's amount remains. Since the initial amount of the radioactive substance was 325 grams, after 1 year,  $325(0.87)$  grams remains; after 2 years  $325(0.87)(0.87) = 325(0.87)^2$  grams remains; and after  $t$  years,  $325(0.87)^t$  grams remains. Therefore, the function  $f(t) = 325(0.87)^t$  models the remaining amount of the substance, in grams, after  $t$  years.

Choice B is incorrect and may result from confusing the amount of the substance remaining with the decay rate. Choices C and D are incorrect and may result from confusing the original amount of the substance and the decay rate.