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3.3 Rules for Differentiation

MathKanection for Videos & Guided Notes

Product Rule

Given $f(x)$ and $g(x)$, where $f(x)$ and $g(x)$ are differentiable functions, then

$$\frac{d}{dx}(f(x)g(x)) = g(x)f'(x) + f(x)g'(x)$$

or $gf' + fg'$

See Khan's proof at: https://youtu.be/L5ErlC0COxI

Example 1: Given $h(x) = (3x+4)(2x^2+5x-1)$, find $h'(x)$.

$h'(x) = (2x^2+5x-1)(3) + (3x+4)(4x+5)$

$= 6x^2+15x-3+12x^2+16x+15x+20$

$\boxed{h'(x) = 18x^2+46x+17}$

Example 2: Given $g(x) = \left(\sqrt[4]{x}+1\right)(4-x^2)$, find $g'(x)$.

$= (x^{1/4}+1)(4-x^2)$

$g'(x) = (4-x^2)(\frac{1}{4}x^{-3/4}) + (x^{1/4}+1)(-2x)$

$= x^{-3/4} - \frac{1}{4}x^{5/4} + -2x^{5/4} - 2x$

$\boxed{g'(x) = -\frac{9}{4}x^{5/4} - 2x + x^{-3/4}}$

$2 + -3/4$

$8/4 - 3/4$

$5/4$

$\frac{1}{4}+1$

$\frac{1}{4}+\frac{4}{4}$

$\frac{5}{4}$

Example 3: Find the equation of the tangent line for $v(t) = (t^2+1)(4t^3-5)$ at $t = 1$

$y - y_1 = m(x - x_1)$

$v - v(1) = v'(1)(t-1)$

$v - -2 = 22(t-1)$

$\boxed{v + 2 = 22(t-1)}$

$v(1) = (1^2+1)(4(1)^3-5)$

$= 2(-1)$

$= -2$

$v'(t) = (4t^3-5)(2t) + (t^2+1)(12t^2)$

$v'(1) = (4(1)^3-5)(2\cdot 1) + (1^2+1)(12\cdot 1^2)$

$= (-1)(2) + (2)(12)$

$v'(1) = 22$

Example 4: Let $h(x) = a(x)w(x)$ and $q(x) = 3a(x) - 2w(x)$. Find $h'(1)$ and $q'(0)$.

x	$a(x)$	$a'(x)$	$w(x)$	$w'(x)$
0	3	-1	2	-7
1	4	-2	5	$\frac{1}{2}$

$h'(x) = w(x)a'(x) + a(x)w'(x)$

$h'(1) = w(1)a'(1) + a(1)w'(1)$

$= 5(-2) + 4(\frac{1}{2})$

$\boxed{h'(1) = -8}$

$q'(x) = 3a'(x) - 2w'(x)$

$q'(0) = 3a'(0) - 2w'(0)$

$= 3(-1) - 2(-7)$

$\boxed{q'(0) = 11}$

Example 5: Find $a''(0)$ where $a(x) = (4x^5 - 7x + 3)(x^2 - 4)$.

$a'(x) = (x^2-4)(20x^4-7) + (4x^5-7x+3)(2x)$

$= 20x^6 - 80x^4 - 7x^2 + 28 + 8x^6 - 14x^2 + 6x$

$= 28x^6 - 80x^4 - 21x^2 + 6x + 28$

2nd derivative

derive again!

$a''(x) = 168x^5 - 120x^3 - 42x + 6$

$a''(0) = 168(0)^5 - 120(0)^3 - 42(0) + 6$

$\boxed{a''(0) = 6}$