

3.3 Rules for Differentiation

MathKanection for Videos & Guided Notes

Product Rule

Given $f(x)$ and $g(x)$, where $f(x)$ and $g(x)$ are differentiable functions, then

$$\frac{d}{dx}(f(x)g(x)) = g(x)f'(x) + f(x)g'(x)$$

or $gf' + fg'$

See Khan's proof at: <https://youtu.be/L5ErlC0COxI>

Example 1: Given $h(x) = (3x + 4)(2x^2 + 5x - 1)$, find $h'(x)$.

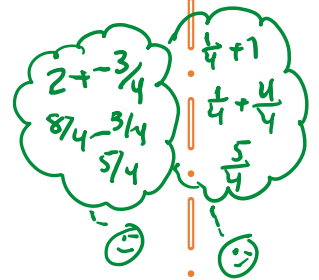
$$\begin{aligned} h'(x) &= (2x^2 + 5x - 1)(3) + (3x + 4)(4x + 5) \\ &= 6x^2 + 15x - 3 + 12x^2 + 16x + 15x + 20 \end{aligned}$$

$$h'(x) = 18x^2 + 46x + 17$$

Example 2: Given $g(x) = (\sqrt[4]{x} + 1)(4 - x^2)$, find $g'(x)$.

$$\begin{aligned} &= (x^{1/4} + 1)(4 - x^2) \\ g'(x) &= (4 - x^2)\left(\frac{1}{4}x^{-3/4}\right) + (x^{1/4} + 1)(-2x) \\ &= x^{-3/4} - \frac{1}{4}x^{5/4} - 2x^{5/4} - 2x \end{aligned}$$

$$g'(x) = -\frac{9}{4}x^{5/4} - 2x + x^{-3/4}$$



Example 3: Find the equation of the tangent line for $v(t) = (t^2 + 1)(4t^3 - 5)$ at $t = 1$

$$y - y_1 = m(x - x_1)$$

$$v - v(1) = v'(1)(t - 1)$$

$$v - -2 = 22(t - 1)$$

$$v + 2 = 22(t - 1)$$

$$\begin{aligned} v(1) &= (1^2 + 1)(4(1)^3 - 5) \\ &= 2(-1) \\ &= -2 \end{aligned}$$

$$v'(t) = (4t^3 - 5)(2t) + (t^2 + 1)(12t^2)$$

$$\begin{aligned} v'(1) &= (4(1)^3 - 5)(2 \cdot 1) + (1^2 + 1)(12 \cdot 1^2) \\ &= (-1)(2) + (2)(12) \end{aligned}$$

$$v'(1) = 22$$

Example 4: Let $h(x) = a(x)w(x)$ and $q(x) = 3a(x) - 2w(x)$. Find $h'(1)$ and $q'(0)$.

x	$a(x)$	$a'(x)$	$w(x)$	$w'(x)$
0	3	-1	2	-7
1	4	-2	5	$\frac{1}{2}$

$$h'(x) = w(x)a'(x) + a(x)w'(x)$$

$$h'(1) = w(1)a'(1) + a(1)w'(1)$$

$$= 5(-2) + 4\left(\frac{1}{2}\right)$$

$$h'(1) = -8$$

$$q'(x) = 3a'(x) - 2w'(x)$$

$$q'(0) = 3a'(0) - 2w'(0)$$

$$= 3(-1) - 2(-7)$$

$$q'(0) = 11$$

Example 5: Find $a''(0)$ where $a(x) = (4x^5 - 7x + 3)(x^2 - 4)$.

$$a'(x) = (x^2 - 4)(20x^4 - 7) + (4x^5 - 7x + 3)(2x)$$

$$= 20x^6 - 80x^4 - 7x^2 + 28 + 8x^6 - 14x^2 + 6x$$

$$= 28x^6 - 80x^4 - 21x^2 + 6x + 28$$

$$a''(x) = 168x^5 - 120x^3 - 42x + 6$$

$$a''(0) = 168(0)^5 - 120(0)^3 - 42(0) + 6$$

$$a''(0) = 6$$

2nd
derivative
again!