3.3

⑤ Product Rule

If $f(x)$ + $g(x)$ are diff'able, then

$$\frac{d}{dx}\left(f(x)\cdot g(x)\right) = g(x)\cdot f'(x) + f(x)g'(x)$$

$$\text{OR} \quad gf' + fg'$$

ex: $h(x) = \underbrace{(3x+4)}_{f}\underbrace{(2x^2+5x-1)}_{g}$

gf' + fg'

$$h'(x) = \underbrace{(2x^2+5x-1)}_{g}\underbrace{(3)}_{f'} + \underbrace{(3x+4)}_{f}\underbrace{(4x+5)}_{g'}$$

$$= 6x^2+15x-3+12x^2+15x+16x+20$$

$$\boxed{h'(x) = 18x^2+46x+17}$$

ex: $g(x) = (\sqrt[4]{x}+1)(4-x^2)$

$$= \underbrace{(x^{1/4}+1)}_{f}\underbrace{(4-x^2)}_{g}$$

gf' + fg'

$$g'(x) = \underbrace{(4-x^2)}_{g}\underbrace{\left(\frac{1}{4}x^{-3/4}\right)}_{f'} +$$

$\frac{1}{4} - 1$

$\frac{1}{4} - \frac{4}{4}$

$$\underbrace{(x^{1/4}+1)}_{f}\underbrace{(-2x)}_{g'}$$

$$g'(x) = (4-x^2)\left(\frac{1}{4}x^{-3/4}\right) + (x^{1/4}+1)(-2x)$$

$$= x^{-3/4} - \frac{1}{4}x^{5/4} - 2x^{5/4} - 2x$$

$\frac{4}{1} \cdot \frac{1}{4}$

$$\boxed{g'(x) = -\frac{9}{4}x^{5/4} + x^{-3/4} - 2x}$$

$\frac{2}{1} + -\frac{3}{4}$

$\frac{8}{4} + -\frac{3}{4}$

$-\frac{1}{4} - 2$

$-\frac{1}{4} - \frac{8}{4}$

$\frac{1}{4} + 1$

$\frac{1}{4} + \frac{4}{4}$

ex: Find the equation of tangent Line @ $t=1$

$$v(t) = (t^2+1)(4t^3-5)$$

$y - y_1 = m(x - x_1)$

$v - v_1 = m(t - t_1)$

$$v'(t) = (4t^3-5)(2t) + (t^2+1)(12t^2)$$

$$v'(1) = (4(1)^3-5)(2(1)) + (1^2+1)(12(1)^2)$$

$$= (-1)(2) + 2(12)$$

$$= -2 + 24 = 22 \leftarrow \text{slope @ } t=1$$

$$v(1) = (1^2+1)(4(1)^3-5)$$

$$= 2(-1) = -2 \leftarrow \text{y-value @ } t=1$$

(v-value @ t = 1)

$$v - v_1 = m(t - t_1)$$

$$\boxed{v + 2 = 22(t-1)}$$

ex: Let $h(x) = f(x)\cdot g(x)$. If $f(1)=4$, $g(1)=5$, $f'(1) = -2$, $g'(1) = \frac{1}{2}$. Find $h'(1)$.

$$h'(x) = g(x)\cdot f'(x) + f(x)\,g'(x)$$

$$h'(1) = g(1)\,f'(1) + f(1)\,g'(1)$$

$$= 5(-2) + 4\left(\tfrac{1}{2}\right)$$

$$= -10 + 2$$

$$\boxed{h'(1) = -8}$$