

Product Rule Practice

For each of the following functions, identify one needs to use the product rule to find the derivative. If yes, then identify the two functions, $f(x)$ and $g(x)$.

	Function	Product Rule		$f(x)$	$g(x)$
		YES	NO		
1	$a(x) = 3e^{2x}$		X		
2	$b(x) = \cos x e^{2x}$	X		$\cos x$	e^{2x}
3	$c(x) = 4 \sin x$		X		
4	$d(x) = 4x \sin x$	X		$4x$	$\sin x$
5	$h(x) = \sin 4x$		X		
6	$j(x) = (4x - 3)(x^2 + 7x - 1)$	X		$4x - 3$	$x^2 + 7x - 1$
7	$m(x) = 4(x - 1)$		X		
8	$p(x) = 4x\sqrt{x}$		X		

☺... $4x\sqrt{x} = 4x \cdot x^{1/2} = 4x^{3/2}$

Product Rule Numerically

x	-1	0	3	4	8
$f(x)$	0	-3	1	-1	4
$f'(x)$	2	6	7	8	0
$g(x)$	5	1	-6	-1	-12
$g'(x)$	-2	3	5	$\frac{1}{2}$	10

Use the table above for problems #9 – 13.

- 9) Given $h(x) = 3xf(x)$, find $h'(0)$.

$$h'(x) = f(x) \cdot 3 + 3x \cdot f'(x)$$

$$h'(0) = f(0) \cdot 3 + 3(0) \cdot f'(0)$$

$$= -3 \cdot 3 + 0$$

$$h'(0) = -9$$

- 10) Given $r(x) = (4\sqrt{x} + 2)g(x)$, find $r'(4)$.

$$r(x) = (4x^{1/2} + 2)g(x)$$

$$r'(x) = g(x) \cdot 2x^{-1/2} + (4x^{1/2} + 2)g'(x)$$

$$r'(4) = g(4) \cdot 2(4)^{-1/2} + (4(4)^{1/2} + 2)g'(4)$$

$$= -1 \cdot 2 \cdot \frac{1}{\sqrt{4}} + (4\sqrt{4} + 2) \cdot \frac{1}{2}$$

$$= -1 + 5$$

$$r'(4) = 4$$

- 11) Given $v(x) = 2g(x)f(x)$, find $v'(3)$.

$$v'(x) = f(x) \cdot 2g'(x) + 2g(x) f'(x)$$

$$\text{or } v'(x) = 2(g(x)f'(x) + f(x)g'(x))$$

$$v'(3) = f(3) \cdot 2g'(3) + 2g(3) f'(3)$$

$$= 1 \cdot 2 \cdot 5 + 2 \cdot -6 \cdot 7$$

$$v'(3) = -74$$

- 12) Given $w(x) = \left(\frac{1}{x} + 3x^2 - 1\right)g(x)$, find $w'(-1)$.

$$w(x) = (x^{-1} + 3x^2 - 1)g(x)$$

$$w'(x) = g(x)(-x^{-2} + 6x) + (x^{-1} + 3x^2 - 1)g'(x)$$

$$\begin{aligned} w'(-1) &= g(-1)\left(-\frac{1}{(-1)^2} + 6(-1)\right) + \left(\frac{1}{-1} + 3(-1)^2 - 1\right)g'(-1) \\ &= 5(-7) + 1(-2) \end{aligned}$$

$$w'(-1) = -37$$

- 13) Write the equation of the line (in point-slope form) tangent to $w(x) = \sqrt[3]{x}g(x)$ at $x = 8$.

$$w(x) = x^{1/3}g(x)$$

$$w'(x) = g(x) \cdot \frac{1}{3}x^{-2/3} + x^{1/3}g'(x)$$

$$\begin{aligned} w'(8) &= g(8) \cdot \frac{1}{3}(8)^{-2/3} + 8^{1/3}g'(8) \\ &= -12 \cdot \frac{1}{3} \cdot \frac{1}{(3\sqrt{8})^2} + \sqrt[3]{8} \cdot 10 \\ &= -12 \cdot \frac{1}{3} \cdot \frac{1}{4} + 2 \cdot 10 \\ &= -1 + 20 \end{aligned}$$

$$y - y_1 = m(x - x_1)$$

$$y - w(8) = w'(8)(x - 8)$$

$$y - 24 = 19(x - 8)$$

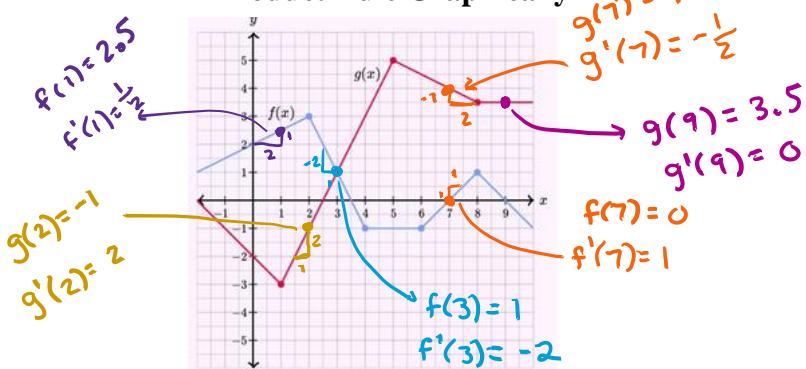
$$y + 24 = 19(x - 8)$$

$$w(8) = \sqrt[3]{8}g(8)$$

$$= 2(-1^2)$$

$$= -24$$

Product Rule Graphically



Use the graph above of the functions f and g for problems #14 – 18.

- 14) Given $h(x) = 4x^2f(x)$. Find the slope of $h(x)$ at $x = 1$.

$$h'(x) = f(x) \cdot 8x + 4x^2f'(x) \quad \text{means } h'(1)$$

$$h'(1) = f(1) \cdot 8(1) + 4(1)^2f'(1)$$

$$= 2.5(8) + 4(1/2)$$

$$h'(1) = 22$$

- 15) Given $y(x) = f(x)g(x)$. Find the slope of the tangent line to $y(x)$ at $x = 7$.

$$y'(x) = g(x)f'(x) + f(x)g'(x) \quad \text{means } y'(7)$$

$$y'(7) = g(7)f'(7) + f(7)g'(7)$$

$$= 4(1) + 0(-1)$$

$$y'(7) = 4$$

- 16) Find the instantaneous rate of change for $d(x)$ at $x = 9$, where $d(x) = \frac{2}{\sqrt{x}}g(x) = 2x^{-1/2}g(x)$

$$d'(x) = g(x) \cdot -\frac{1}{2}x^{-3/2} + 2x^{-1/2}g'(x)$$

$$\begin{aligned} d'(9) &= g(9) \cdot -\frac{1}{2}(\sqrt{9})^{-3} + 2 \cdot \frac{1}{\sqrt{9}} \cdot g'(9) \\ &= 3.5 \cdot -\frac{1}{2} \cdot \frac{1}{27} + 2 \cdot \frac{1}{3} \cdot 0 \end{aligned}$$

$$\begin{aligned} &= \frac{7}{2} \cdot -\frac{1}{2} \cdot \frac{1}{27} \\ d'(9) &= -\frac{7}{108} \end{aligned}$$

- 17) Write the equation of the line (in point-slope form) tangent to the curve $p(x) = (x^3 - 2)f(x)$ at $x = 3$.

$$y - y_1 = m(x - x_1)$$

$$p'(x) = f(x) \cdot 3x^2 + (x^3 - 2)f'(x)$$

$$p'(3) = f(3) \cdot 3(3)^2 + (3^3 - 2)f'(3)$$

$$= 1 \cdot 27 + 25(-2)$$

$$= 27 - 50$$

$$p'(3) = -23$$

$$y - p(3) = p'(3)(x - 3)$$

$$y - 25 = -23(x - 3)$$

$$p(3) = (3^3 - 2)f(3)$$

$$= (25)(1)$$

$$p(3) = 25$$

- 18) Given $v(x) = g(x) \left(\frac{3}{2}x^4 + 4x - 1\right)$, find $v'(2)$.

$$v'(x) = \left(\frac{3}{2}x^4 + 4x - 1\right)g'(x) + g(x) \cdot (6x^3 + 4)$$

$$v'(2) = \left(\frac{3}{2}(2)^4 + 4(2) - 1\right)g'(2) + g(2)(6(2)^3 + 4)$$

$$= (31)(2) + -1(52)$$

$$v'(2) = 10$$

Product Rule Analytically (Algebraically)

- 19) Find the derivative of the function $f(x) = (x^3 - 2x + 1)(x - 3)$ using the product rule.

$$f'(x) = (x-3)(3x^2 - 2) + (x^3 - 2x + 1)(1)$$

$$f'(x) = 3x^3 - 9x^2 - 2x + 6 + x^3 - 2x + 1$$

$$f'(x) = 4x^3 - 9x^2 - 4x + 7$$

- 20) Find the derivative of the function $f(x) = (x^3 - 2x + 1)(x - 3)$ by distributing first. Verify that the answer is the same as #19.

checks, is the same ✓

$$f(x) = x^4 - 2x^3 + x - 3x^3 + 6x - 3$$

$$= x^4 - 3x^3 - 2x^2 + 7x - 3$$

$$f'(x) = 4x^3 - 9x^2 - 4x + 7$$