3.3

⑥ Quotient Rule

If $f(x)$ + $g(x)$ are diff'able and $\frac{f(x)}{g(x)}$ is diff'able,

$$\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) = \frac{g(x)f'(x) - f(x)g'(x)}{(g(x))^2}$$

or $\frac{gf' - fg'}{g^2}$

$$\frac{d}{dx}\left(\frac{hi}{lo}\right) = \frac{lo\ dhi - hi\ dlo}{(lo)^2}$$

ex: $h(x) = \frac{x^2+1}{x-4}$ $\begin{matrix} hi \\ lo \end{matrix}$

$$\frac{lo\ dhi - hi\ dlo}{(lo)^2}$$

$$h'(x) = \frac{(x-4)(2x) - (x^2+1)(1)}{(x-4)^2}$$

$$= \frac{2x^2 - 8x - x^2 - 1}{(x-4)^2}$$

$$\boxed{h'(x) = \frac{x^2 - 8x - 1}{(x-4)^2}}$$

ex: Find $\left.\frac{d^2y}{dx^2}\right|_{x=2}$ when $y = \frac{1+x^3}{x^2}$ $\begin{matrix} hi \\ lo \end{matrix}$

2nd derivative @ $x=2$

$\frac{dy}{dx} \quad \frac{d^2y}{dx^2}$

$$\frac{dy}{dx} = \frac{x^2(3x^2) - (1+x^3)(2x)}{(x^2)^2}$$

$\frac{lo\, dhi - hi\, dlo}{lo^2}$

$$= \frac{3x^4 - 2x - 2x^4}{x^4}$$

$$= \frac{x^4 - 2x}{x^4}$$

$$= \frac{\cancel{x}(x^3 - 2)}{\cancel{x}(x^3)}$$

$$\frac{dy}{dx} = \frac{x^3 - 2}{x^3} \quad \begin{matrix} hi \\ lo \end{matrix}$$

$$\frac{d^2y}{dx^2} = \frac{x^3(3x^2) - (x^3-2)(3x^2)}{(x^3)^2}$$

$\frac{lo\, dhi - hi\, dlo}{lo^2}$

$$= \frac{3x^5 - 3x^5 + 6x^2}{x^6}$$

$$= \frac{6x^2}{x^6} = \frac{6}{x^4}$$

$$\left.\frac{d^2y}{dx^2}\right|_{x=2} = \frac{6}{(2)^4}$$

$$= \frac{6}{16}$$

$$= \boxed{\frac{3}{8}}$$

ex:

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
0	-1	3	7	2
1	4	0	6	$1/4$

Find $H'(1)$ if $H(x) = \dfrac{f(x)}{g(x)} \; \dfrac{f}{g}$

$$H'(x) = \frac{g(x)f'(x) - f(x)g'(x)}{(g(x))^2}$$

$$\frac{gf' - fg'}{g^2}$$

$$H'(1) = \frac{g(1)f'(1) - f(1)g'(1)}{(g(1))^2}$$

$$= \frac{6(0) - 4(1/4)}{(6)^2}$$

$$\boxed{H'(1) = \frac{-1}{36}}$$