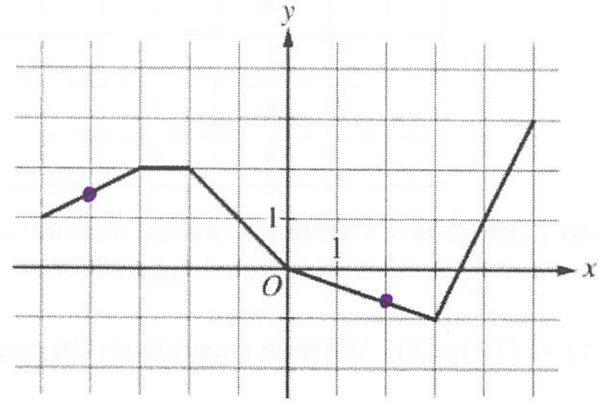
DATE:___________

2017 AP CALCULUS AB FREE-RESPONSE QUESTIONS

x	$g(x)$	$g'(x)$
–5	10	–3
–4	5	–1
–3	2	4
–2	3	1
–1	1	–2
0	0	–3



Graph of h

6. Let f be the function defined by $f(x) = -x^2 + 4x$. Let g be a differentiable function. The table above gives values of g and g' at selected values of x. Let h be the function whose graph, consisting of five line segments, is shown in the figure above.

(a) Find the slope of the line tangent to the graph of f at $x = 1$.

(b) Let k be the function defined by $k(x) = h(x)f(x)$. Find $k'(2)$.

(c) Let m be the function defined by $m(x) = \frac{g(x)}{h(x)}$. Find $m'(-4)$.

(d) Is there a number c in the closed interval $[-5,0]$ such that $g(c) = 4$? Justify your answer.

(d) g is diff'able, so g is cont.

$g(-4)=5$, $g(-3)=2$ } 4 is btn 5 + 2

∴, there is a # c on $[-5,0]$ s.t. $g(c)=4$ b/c g cont on $[-5,0]$ and $g(-4)<4<g($

a) ~~$y-y_1=m(x-x_1)$~~

~~$\boxed{y-3=2(x-1)}$~~

~~$f(1)=-(1)^2+4(1)$~~
$= -1+4$
$= 3$

$f'(x)=-2x+4$
$f'(1)=-2(1)+4$
$\boxed{f'(1)=2}$

b) $k(x)=h(x)f(x)$

$k'(x)= f(x)h'(x) + f'(x)h(x)$

$k'(2)= f(2)h'(2) + f'(2)h(2)$

$= 4(-1/3) + 0(-2/3)$

$\boxed{k'(2)=-4/3}$

$f(2)=-(2)^2+4(2)$
$= -4+8$
$= 4$

$h(2)= -2/3$

$f'(2)=-2(2)+4$
$=0$

$h'(2)=-\frac{1}{3}$ $\frac{\text{rise}}{\text{run}}$

$h'(-4)=\frac{1}{2}$ $\frac{\text{rise}}{\text{run}}$

c) $m'(x)=\dfrac{h(x)g'(x)-g(x)h'(x)}{\cancel{(g(x))^2}\,(h(x))^2}$

$m'(-4)=\dfrac{h(-4)g'(-4)-g(-4)h'(-4)}{\cancel{(g(-4))^2}\,(h(-4))^2} \Rightarrow \dfrac{(3/2)(-1)-5(\frac{1}{2})}{\cancel{(5)^2}\,(3/2)^2}$

$=\dfrac{-3/2-5/2}{\cancel{25}\;9/4} = \boxed{\cancel{\frac{-4}{25}}}\ \boxed{\frac{-16}{9}}$

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
1	−6	3	2	8
2	2	−2	−3	0
3	8	7	6	2
6	4	5	3	−1

6. The functions f and g have continuous second derivatives. The table above gives values of the functions their derivatives at selected values of x.

(a) Let $k(x) = f(x)g(x)$. Write an equation for the line tangent to the gaph of k at $x = 3$.

(b) Let $h(x) = \frac{g(x)}{f(x)}$. Find $h'(1)$.

a) $k'(x) = g(x)f'(x) + f(x)g'(x)$

$k'(3) = g(3)f'(3) + f(3)g'(3)$

$= 6(7) + 8(2)$

$= 42 + 16$

$= 58$

$k(3) = f(3)g(3)$

$= 8 \cdot 6$

$= 48$

$\boxed{y - 48 = 58(x-3)}$

b) $h'(x) = \dfrac{f(x)g'(x) - g(x)f'(x)}{(f(x))^2}$

$h'(1) = \dfrac{f(1)g'(1) - g(1)f'(1)}{(f(1))^2}$

$= \dfrac{-6 \cdot 8 - 2(3)}{(-6)^2}$

$= \dfrac{-48 - 6}{36}$

$= \dfrac{-54}{36}$

$= \dfrac{-6}{4}$

$\boxed{h'(1) = \dfrac{-3}{2}}$