DATE:_____

Particle Motion Practice

1. A particle moves along the x-axis so that at any time $t \geq 0$ its position is given by $x(t) = t^3 - 12t + 5$.

a) What is the particle's initial position? → $t=0$

$x(0) = 0^3 - 12(0) + 5$

$\boxed{x(0) = 5}$

b) What is the average velocity over the time interval [1,4]? Show the computations that lead to your answer. → $\frac{\Delta x}{\Delta t}$

$v_{avg} = \frac{x(4) - x(1)}{4-1}$

$= \frac{4^3 - 12(4) + 5 - (1^3 - 12(1) + 5)}{3}$

$= \frac{64 - 48 + 5 - 1 + 12 - 5}{3}$

$= \frac{27}{3} = \boxed{9}$

c) At time $t = 4$, is the speed of the particle increasing or decreasing? Explain your answr.

↳ $v(t)$ and $a(t)$ same signs ; different signs

$x(t) = t^3 - 12t + 5$

$v(t) = x'(t) = 3t^2 - 12 \longrightarrow v(4) = 3(4)^2 - 12 > 0$

$a(t) = x''(t) = 6t \longrightarrow a(4) = 6(4) > 0$

Speed of the particle is increasing @ $t=4$ b/c $v(4) > 0$ and $a(4) > 0$ (velocity & acceleration have the same signs)

2.

t (sec)	0	3	5	8	12	14	17	20	25
$v(t)$ (ft/sec)	15	9	6	4	2	−3	−5	−8	−14

The table above provides the velocities of a rocket recorded at specific times. Using the table, answer the following questions:

a) Is there ever an interval in which velocity of the rocket is zero? Explain your answer.

↳ IVT

$v(t)$ is continuous b/c velocity is continuous.

$v(12) = 2$
$v(14) = -3$
} 0 is btn 2 and −3

∴, by IVT, there is a time interval in which velocity is zero

$\exists c$ on $[12,14]$, $v(14) < 0 < v(12)$.

2.

t (sec)	0	3	5	8	12	14	17	20	25
$v(t)$ (ft/sec)	15	9	6	4	2	−3	−5	−8	−14

The table above provides the velocities of a rocket recorded at specific times. Using the table, answer the following questions:

b) During which time interval is the rocket's position decreasing? Explain your answer.

↳ $v(t) < 0$

on $[14, 25]$, rocket's position is decreasing b/c $v(t) < 0$ on $[14, 25]$

c) Find an approximation for the acceleration of the rocket at $t = 6$. Show the computations that lead to your answer.

→ $\frac{\Delta v}{\Delta t}$

$$a(6) = v'(6) \approx \frac{v(8) - v(5)}{8 - 5}$$

$$= \frac{4 - 6}{3}$$

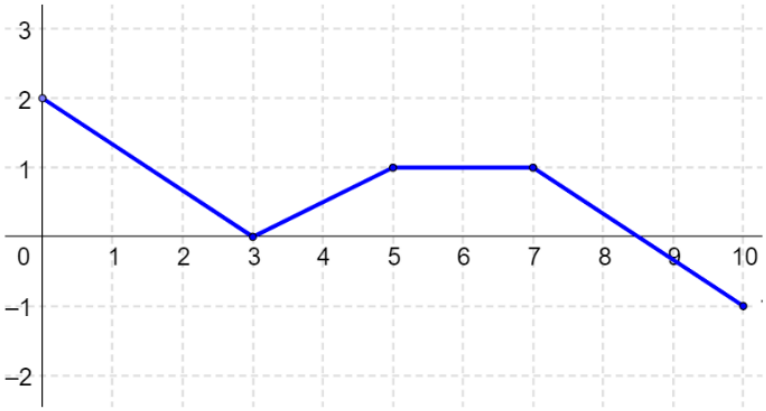
$$a(6) \approx -\frac{2}{3} \text{ ft/sec}^2$$

units:

$\frac{\Delta v}{\Delta t} \rightarrow \frac{\text{ft/sec}}{\text{sec}} = \frac{\text{ft}}{\text{sec}} \cdot \frac{1}{\text{sec}}$

$= \frac{\text{ft}}{\text{sec}^2}$

3. A particle moves along the x-axis with velocity as shown in the graph below.



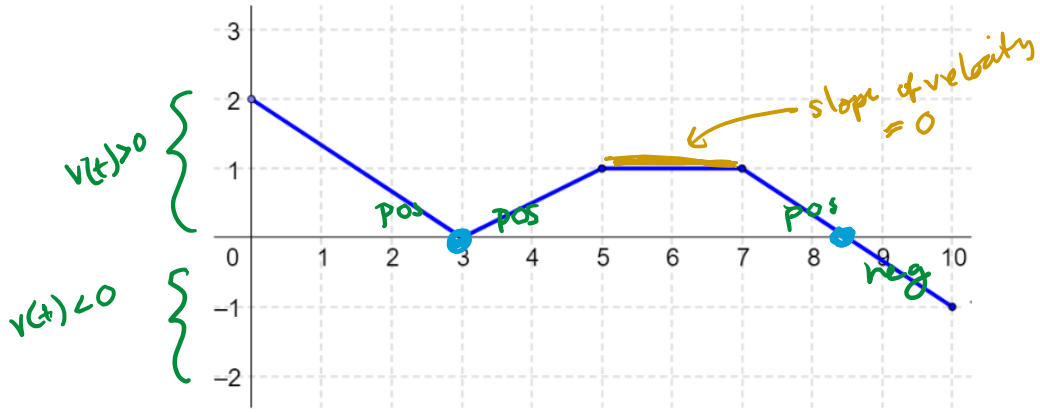
a) At $t = 0$, is the particle moving to the left or right? Justify your answer.

↳ $v < 0$ $\quad$ $v > 0$

$v(0) = 2 > 0$

Particle is moving to the right @ $t = 0$ b/c $v(0) > 0$

3. A particle moves along the x-axis with velocity as shown in the graph below.



b) When is the particle at rest? Justify your answer.

↳ v = 0

The particle is at rest @ t = 3 and t = 8.5 b/c v(3) = 0 and v(8.5) = 0.

c) When is the acceleration of the particle zero? Justify your answer.

↳ a(t) = v'(t) → slope of velocity

a(t) = 0 on (5,7) b/c velocity has horizontal tangent lines on (5,7)

d) When does the particle change direction? Justify your answer.

↳ velocity changes signs

Particle changes direction @ t = 8.5 b/c v(t) changes from pos. to neg. @ t = 8.5