

**3.4 Properties of Logarithmic Functions**

Target 3C: Understand properties of common and natural logarithmic functions  
Target 3E: Know and apply product, quotient and power rules of logarithmic functions

**SAT Connection****Passport to Advanced Math**

8. Solve a system of one linear equation and one quadratic equation.

Example: If  $\frac{x^{a^2}}{x^{b^2}} = x^{16}$ ,  $x > 1$ , and  $a + b = 2$ , what is the value

of  $a - b$  ?

- A) 8
- B) 14
- C) 16
- D) 18

Solution**Properties of Logs/Natural Logs**

Product Property:  $\log_b(xy) =$

$$\ln(xy) =$$



Quotient Property:  $\log_b\left(\frac{x}{y}\right) =$

$$\ln\left(\frac{x}{y}\right) =$$

Power Property:  $\log_b x^c =$

$$\ln x^c =$$

Change of Base:  $\log_b x = \frac{\log_a x}{\log_a b}$

$$\log_b x = \frac{\ln x}{\ln b}$$

*Examples*

Using the properties of logarithms, expand the logarithmic expression.

1.  $\ln 3x$

2.  $\log\left(\frac{4x}{y^2}\right)$

3.  $\log_2 25x^3$

4.  $\log\sqrt[3]{\frac{x^2}{y}}$

Using the properties of logarithms, condense the logarithms into a single expression.

5.  $\log x + 3 \log y$

6.  $\ln 4x - \ln 2y$

7.  $2 \log x - \frac{1}{3} \log y + \log a$

Write the expression as a natural logarithm.

8.  $\log_5 x$

9.  $\log_4(2x + y)$

**More Practice****Properties of Logarithms**

<https://www.khanacademy.org/math/algebra2/exponential-and-logarithmic-functions/properties-of-logarithms/v/introduction-to-logarithm-properties>

[http://www.algebralab.org/lessons/lesson.aspx?file=algebra\\_logarithmproperties.xml](http://www.algebralab.org/lessons/lesson.aspx?file=algebra_logarithmproperties.xml)

<http://www.regentsprep.org/regents/math/algtrig/ate9/LogPrac.htm>

<http://www.mathguide.com/lessons2/Logs.html>

<https://www.youtube.com/watch?v=SxF44olWTyk>

<https://www.youtube.com/watch?v=eLapHtvQbFo>

**Homework Assignment**

p.317 #1-21 odd,29,31,52,53

**SAT Connection**  
**Solution**

**Choice A is correct.** Since the numerator and denominator of  $\frac{x^{a^2}}{x^{b^2}}$  have a common base, it follows by the laws of exponents that this expression can be rewritten as  $x^{a^2 - b^2}$ . Thus, the equation  $\frac{x^{a^2}}{x^{b^2}} = 16$  can be rewritten as  $x^{a^2 - b^2} = x^{16}$ . Because the equivalent expressions have the common base  $x$ , and  $x > 1$ , it follows that the exponents of the two expressions must also be equivalent. Hence, the equation  $a^2 - b^2 = 16$  must be true. The left-hand side of this new equation is a difference of squares, and so it can be factored:  $(a + b)(a - b) = 16$ . It is given that  $(a + b) = 2$ ; substituting 2 for the factor  $(a + b)$  gives  $2(a - b) = 16$ . Finally, dividing both sides of  $2(a - b) = 16$  by 2 gives  $a - b = 8$ .

Choices B, C, and D are incorrect and may result from errors in applying the laws of exponents or errors in solving the equation  $a^2 - b^2 = 16$ .