3.5 Equation Solving & Modeling

Target 3B: Know and understand the inverse relationships of exponential and logarithmic equations

SAT Connection

Problem Solving and Data Analysis

4. Given a scatterplot, use linear, quadratic, or exponential models to describe how the variables are related.

Example: The population of mosquitoes in a swamp is estimated over the course of twenty weeks, as shown in the table.

Time (weeks)	Population
0	100
5	1,000
10	10,000
15	100,000
20	1,000,000

exponential

$t = 0, P(0) = 100$

$t = 5, P(5) = 100(10)$

$t = 10, P(10) = 100(10)^2$

$t = 15, P(15) = 100(10)^3$

$t = 20, P(20) = 100(10)^4$

$\vdots$

$P(t) = 100(10)^{t/5}$

Which of the following best describes the relationship between time and the estimated population of mosquitoes during the twenty weeks?

A) Increasing linear

B) Decreasing linear

C) Exponential growth

D) Exponential decay

Solution

Solving Exponential & Logarithmic Equations

Watch a video or read a website, then solve the following problems.

http://www.regentsprep.org/regents/math/algtrig/ate8/exponentialEquations.htm

https://www.youtube.com/watch?v=M6f6dANVyxA

Find the solution algebraically.

1. $32\left(\frac{1}{4}\right)^{\frac{x}{3}} = 2$

$\left(\frac{1}{4}\right)^{x/3} = \frac{2}{32}$ * divide by 32

$\left(\frac{1}{4}\right)^{x/3} = \frac{1}{16}$ * reduce

$(4^{-1})^{x/3} = 4^{-2}$ * rewrite w/ base 4

$4^{-x/3} = 4^{-2}$

$-\frac{x}{3} = -2$ * bases =, so exponents =

$-x = -6$

$x = 6$

2. $3 \cdot 2^x = 48$ * divide by 3

$2^x = 16$ * rewrite w/ base 2

$2^x = 2^4$ * bases =, exp. =

$x = 4$

3. $0.35^x = 8$

$\ln 0.35^x = \ln 8$ * log/ln both side

$x \ln 0.35 = \ln 8$ * power proper

$x = \frac{\ln 8}{\ln 0.35}$ * divide

$x = -1.981$

4. $2 \cdot 10^{2x} = 14$

$10^{2x} = 7$ * divide by 2

$\log 10^{2x} = \log 7$ * log both sides

$2x = \log 7$ * divide by 2

$x = \frac{\log 7}{2}$

$x = 0.423$

5. $3 + 2e^{-x} = 6$

$2e^{-x} = 3$ * subtract 3

$e^{-x} = 3/2$ * divide by 2

$\ln e^{-x} = \ln(\frac{3}{2})$ * ln both sides

$-x = \ln(\frac{3}{2})$ * divide by -1

$x = -\ln(\frac{3}{2})$

$x = -.405$

Watch a video or read a website, then solve the following problems.
http://www.regentsprep.org/regents/math/algtrig/ate9/logseq.htm
https://youtu.be/59j0ALU3N7k

6. $3\ln(x-4) - 2 = 6$

$3\ln(x-4) = 8$ * add 2

$\ln(x-4) = \frac{8}{3}$ * divide by 3

$e^{\ln(x-4)} = e^{8/3}$ * antilog "e" both sides

$x - 4 = e^{8/3}$ * add 4

$x = e^{8/3} + 4$

$x = 18.392$

7. $\log x - \log(x+4) = 1$

$\log(\frac{x}{x+4}) = 1$ * quotient property

$10^{\log(\frac{x}{x+4})} = 10^1$ * antilog "10" both sides

$\frac{x}{x+4} = 10$ * multiply by $x+4$

$x = 10x + 40$ * solve for x

$-9x = 40$

$x = -\frac{40}{9}$ ← extraneous solution

check

$\log(-\frac{40}{9}) - \log(-\frac{40}{9}+4) = 1$

can't take log of - #

∴ no solution

8. $\ln(3x-2) + \ln(x-1) = 2\ln x$

$\ln((3x-2)(x-1)) = \ln x^2$ * product * power

$e^{\ln((3x-2)(x-1))} = e^{\ln x^2}$ * antilog "e" both sides

$(3x-2)(x-1) = x^2$

$3x^2 - 3x - 2x + 2 = x^2$

$3x^2 - 5x + 2 = x^2$

$2x^2 - 5x + 2 = 0$

$2x^2 - 4x - 1x + 2 = 0$

$2x(x-2) - 1(x-2) = 0$

$(2x-1)(x-2) = 0$

$x = 1/2$, $x = 2$

extraneous

$\ln(3\cdot\frac{1}{2} - 2) + \ln(\frac{1}{2} - 1) = 2\ln\frac{1}{2}$

$\ln(-\frac{1}{2}) + \ln(-\frac{1}{2})$

can't take ln of - # ... not in domain of ln

9. $\frac{2^x + 5\cdot 2^{-x}}{2} = 3$

$2^x + 5\cdot 2^{-x} = 6$ * multiply by 2

$2^x + 5(\frac{1}{2^x}) = 6$ * recriate neg exp.

$2^x \cdot 2^x + 5(\frac{1}{2^x})\cdot 2^x = 6\cdot 2^x$ * multiply by LCD

$(2^x)^2 + 5 = 6\cdot 2^x$

$(2^x)^2 - 6\cdot 2^x + 5 = 0$

let $u = 2^x$

$u^2 - 6u + 5 = 0$

$(u-5)(u-1) = 0$

$u = 5$, $u = 1$

$2^x = 5$ $\quad 2^x = 1$ * replace back 2^x for u

$\ln 2^x = \ln 5$ $\quad \ln 2^x = \ln 1$ * solve for x

$x\ln 2 = \ln 5$ $\quad x\ln 2 = 0$

$x = \frac{\ln 5}{\ln 2}$ $\quad x = 0$

$x = 2.322$

More Practice

Solving Exponential Functions

http://www.regentsprep.org/regents/math/algtrig/ate8/exponentialEquations.htm
http://www.purplemath.com/modules/solvexpo2.htm
http://www.sosmath.com/algebra/logs/log4/log46/log46.html
http://www.coolmath.com/algebra/17-exponentials-logarithms/11-solving-exponential-equations-01
https://www.youtube.com/watch?v=M6f6dANVyxA
https://www.youtube.com/watch?v=5R5mKpLsfYg

Solving Logarithmic Functions

http://www.regentsprep.org/regents/math/algtrig/ate9/logseq.htm
http://www.coolmath.com/algebra/17-exponentials-logarithms/15-solving-logarithmic-equations-01
http://www.mesacc.edu/~scotz47781/mat120/notes/logarithms/solving/solving_log_eqns_intro.pdf
https://youtu.be/59j0ALU3N7k
https://www.youtube.com/watch?v=UpE9snwzp30

Homework Assignment

p.331 #1,3,6,11,13,16,17,27, 29,35,37

SAT Connection

Solution

Choice C is correct. The mosquito population starts at 100 in week 0 and then is multiplied by a factor of 10 every 5 weeks. Thus, if $P(t)$ is the mosquito population after t weeks, then based on the table, $P(t) = 100(10)^{\frac{t}{5}}$, which indicates an exponential growth relationship.

Choices A, B, and D are incorrect and may be the result of an incorrect interpretation of the relationship or errors in modeling the relationship.