Date: ____________

Derivatives of Trigonometric Functions

1. If $f(x) = \sin x$, then $f'\left(\frac{\pi}{3}\right) =$

(A) $-\frac{1}{2}$

$\boxed{\text{(B)}}$ $\frac{1}{2}$

(C) $\frac{\sqrt{2}}{2}$

(D) $\frac{\sqrt{3}}{2}$

$f(x) = \sin x$

$f'(x) = \cos x$

$f'(\pi/3) = \cos \frac{\pi}{3}$

$= \boxed{\frac{1}{2}}$

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2. The $\lim\limits_{h \to 0} \frac{\tan(\pi+h) - \tan(\pi)}{h}$ is:

↳ definition of derivative where $f(x) = \tan x$ and $x = \pi$

(A) -1

(B) 0

$\boxed{\text{(C)}}$ 1

(D) does not exist

$f'(x) = \sec^2 x$

$f'(\pi) = \sec^2 \pi = (\sec \pi)^2 = \left(\frac{1}{\cos \pi}\right)^2$

$= \left(\frac{1}{-1}\right)^2 = \boxed{1}$

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3. If $y = \sec x$, then $\frac{d^2y}{dx^2} =$ → 2nd derivative

(A) $\sec x \tan x$

(B) $\sec^3 x \tan x$

(C) $\sec x \tan x + \sec^2 x$

$\boxed{\text{(D) } \sec x \tan^2 x + \sec^3 x}$

$y = \sec x$

$\frac{dy}{dx} = \sec x \tan x$ → product rule 😊

$\frac{d^2y}{dx^2} = \tan x(\sec x \tan x) + \sec x(\sec^2 x)$

$= \sec x \tan^2 x + \sec^3 x$

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4. Given $f(x) = \cos x$ and $g(x) = x^2 + 3x$, if $h(x) = f(x)g(x)$, find $h'(x)$.

↳ product 😊

$h(x) = (\cos x)(x^2 + 3x)$

$h'(x) = (x^2+3x)(-\sin x) + (\cos x)(2x+3)$

$\boxed{h'(x) = -(x^2+3x)\sin x + (2x+3)\cos x}$

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5. Given the velocity of a particle is $v(t) = \cos t$ on the interval $[0, 2\pi)$, when is the particle speeding up?

↳ $v(t)$ + $a(t)$ have same signs

$v(t) = \cos t$ $a(t) = -\sin t$

$\cos t > 0$ on $(0, \pi/2) \cup (3\pi/2, 2\pi)$ and $-\sin t > 0$, $\sin t < 0$ on $(\pi, 2\pi)$ → $(3\pi/2, 2\pi)$

$\cos t < 0$ on $(\pi/2, 3\pi/2)$ and $-\sin t < 0$, $\sin t > 0$ on $(0, \pi)$ → $(\pi/2, \pi)$

Particle speeding up on $(\pi/2, \pi) \cup (3\pi/2, 2\pi)$ b/c $a(t)$ + $v(t)$ have same signs on $(\pi/2, \pi) \cup (3\pi/2, 2\pi)$