3.5 Trig Derivatives

$\frac{d}{dx}(\sin x) = \cos x$ $\quad$ $\frac{d}{dx}(\cos x) = -\sin x$

$\frac{d}{dx}(\tan x) = \sec^2 x$ $\quad$ $\frac{d}{dx}(\cot x) = -\csc^2 x$

$\frac{d}{dx}(\sec x) = \sec x \tan x$ $\quad$ $\frac{d}{dx}(\csc x) = -\csc x \cot x$

ex: $f(x) = \underbrace{x}_{f}\underbrace{\cos x}_{g} + 6$ $\quad$ product rule $gf' + fg'$

$f'(x) = (\cos x)(1) + x(-\sin x) + 0$

$= \cos x - x\sin x$

ex: $f(x) = \underbrace{x^2}_{f}\underbrace{\sin x}_{g} + \cos x$ $\quad$ product rule $gf' + fg'$

$f'(x) = (\sin x)(2x) + x^2(\cos x) + -\sin x$

$= 2x\sin x + x^2\cos x - \sin x$

ex: $g(x) = \dfrac{x}{\tan x}\begin{matrix}hi\\lo\end{matrix} + 3x$

quotient rule: $\dfrac{lo\ dhi - hi\ dlo}{lo^2}$

$$g'(x) = \frac{(\tan x)(1) - x(\sec^2 x)}{(\tan x)^2} + 3$$

$$= \frac{\tan x - x\sec^2 x}{\tan^2 x} + 3$$

ex: $h(x) = \dfrac{\cot x}{3} - 3x\cot x$

$$= \tfrac{1}{3}\cot x - \underbrace{3x}_{f}\underbrace{\cot x}_{g}$$

product rule: $gf' + fg'$

$$h'(x) = \tfrac{1}{3}(-\csc^2 x) - \left[(\cot x)(3) + (3x)(-\csc^2 x)\right]$$

$$= -\tfrac{1}{3}\csc^2 x - 3\cot x + 3x\csc^2 x$$

ex: $f(x) = \dfrac{\sec x}{1+\sec x}\begin{matrix}hi\\lo\end{matrix}$

quotient rule: $\dfrac{lo\ dhi - hi\ dlo}{Lo^2}$

$$f'(x) = \frac{(1+\sec x)(\sec x\tan x) - (\sec x)(\sec x\tan x)}{(1+\sec x)^2}$$

$$= \frac{\sec x \tan x + \sec^2 x \tan x - \sec^2 x \tan x}{(1+\sec x)^2}$$

$$= \frac{\sec x \tan x}{(1+\sec x)^2}$$