3.6 Mathematics of Finance
Target 3G: Solve real-world applications using exponential and logarithmic functions

SAT Connection
Passport to Advanced Math
10. Interpret parts of nonlinear expressions in terms of their context

Example:

Jessica opened a bank account that earns 2 percent interest compounded annually. Her initial deposit was $100, and she uses the expression $100(x)^t$ to find the value of the account after t years.
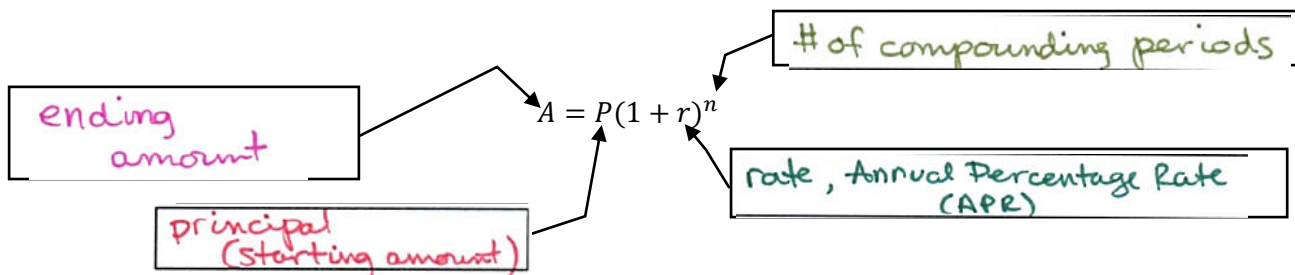
Jessica's friend Tyshaun found an account that earns 2.5 percent interest compounded annually. Tyshaun made an initial deposit of $100 into this account at the same time Jessica made a deposit of $100 into her account. After 10 years, how much more money will Tyshaun's initial deposit have earned than Jessica's initial deposit? (Round your answer to the nearest cent and ignore the dollar sign when gridding your response.)

NOTE: You may start your answers in any column, space permitting. Columns you don't need to use should be left blank.

Jessica

1 year→ $100 + .02(100)$
$100(1.02)$
$\$102.00$

10 years→ $100(1.02)^{10}$
$\$121.90$

Tyshaun

$100 + .025(100)$
$100(1.025)$
$\$102.50$

$100(1.025)^{10}$
$\$128.01$

Solution

Difference: $\$128.01 - \$121.90$
$\$6.11$

Compound Interest (Simple Interest)

ending amount

# of compounding periods

$$A = P(1 + r)^n$$

rate, Annual Percentage Rate (APR)

principal (starting amount)

Example
Find the value of your investment if you invest $500 at 3% compounded annually for 5 years.

$$A = 500(1 + .03)^5$$
$$A = \boxed{\$579.64}$$

- What if you invested this $500 semiannually?

twice a year
so, $n = 2(5)$
$= 10$

← twice a year
so rate $= \frac{.03}{2}$

$A = 500\left(1 + \frac{.03}{2}\right)^{2(5)}$

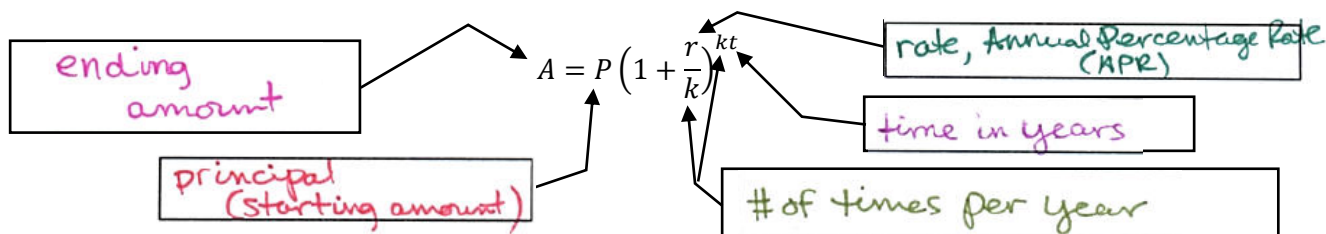$= \boxed{\$580.27}$

- What if you invested this $500 quarterly?

4 times a year
so, $n = 4(5)$
$= 20$

← 4 times a year,
so rate $= \frac{.03}{4}$

$A = 500\left(1 + \frac{.03}{4}\right)^{4(5)}$

$= \$580.59$

Compound Interest (semiannually, quarterly, monthly, daily, periodically, etc.)

ending amount

$$A = P\left(1 + \frac{r}{k}\right)^{kt}$$

rate, Annual Percentage Rate (APR)

time in years

principal (starting amount)

# of times per year

Example
Find the value of your investment if you invest $500 at 3% compounded daily for 5 years.

$A = 500\left(1 + \frac{.03}{365}\right)^{365(5)}$

$A = \boxed{\$580.91}$

$P = 500$
$K = 365$ (365 days in a year)
$r = 3\% = .03$
$t = 5$

- What happens when $k \to \infty$?

$A = \lim\limits_{k \to \infty} P\left(1 + \frac{r}{k}\right)^{kt}$

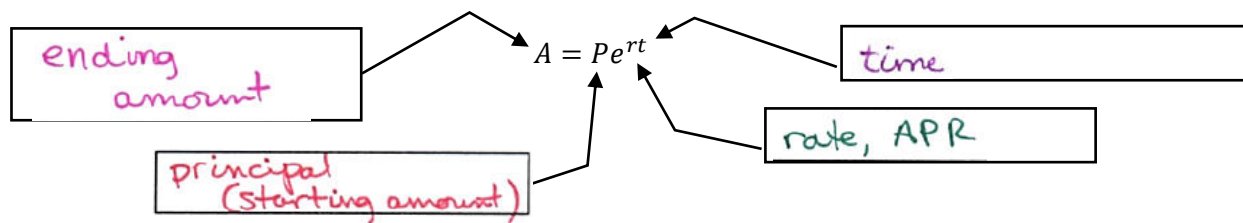$= P\left(\lim\limits_{k \to \infty} \left(1 + \frac{r}{k}\right)^k\right)^t$

$= P(e^r)^t$

$= Pe^{rt}$

let $r = 1$, then $\lim\limits_{k \to \infty}\left(1 + \frac{1}{k}\right)^k = 2.718$
$= e$

if $r = 2$, then $\lim\limits_{k \to \infty}\left(1 + \frac{2}{k}\right)^k = e^2$

$r = 3$, then $\lim\limits_{k \to \infty}\left(1 + \frac{3}{k}\right)^k = e^3$

Compound Continuously

ending amount

principal (starting amount)

$A = Pe^{rt}$

time

rate, APR

Example

Find the value of your investment if you invest $500 at 3% compounded continuously for 5 years.

$A = Pe^{rt}$

$A = 500e^{.03(5)}$

$A = \boxed{\$580.92}$

$P = \$500$
$r = .03$
$t = 5$

More Practice

Compound Periodically

http://www.mathsisfun.com/money/compound-interest-periodic.html
https://www.youtube.com/watch?v=3az4AKvUmmI

Compound Continuously

http://www.coolmath.com/algebra/17-exponentials-logarithms/04-continuous-compound-interest-02
https://www.youtube.com/watch?v=vyJM1n0AvZI

Homework Assignment
p.342 #1,8,11,21,23

The correct answer is 6.11. Jessica made an initial deposit of $100 into her account. The interest on her account is 2 percent compounded annually, so after 10 years, the value of her initial deposit has been multiplied 10 times by the factor $1 + 0.02 = 1.02$. Hence, after 10 years, Jessica's deposit is worth $100(1.02)^{10} = \$121.899$ to the nearest tenth of a cent. Tyshaun made an initial deposit of $100 into his account. The interest on his account is 2.5 percent compounded annually, so after 10 years, the value of his initial deposit has been multiplied 10 times by the factor $1 + 0.025 = 1.025$. Hence, after 10 years, Tyshaun's deposit is worth $100(1.025)^{10} = \$128.008$ to the nearest tenth of a cent. Hence, Jessica's initial deposit earned $21.899 and Tyshaun's initial deposit earned $28.008. Therefore, to the nearest cent, Tyshaun's initial deposit earned $6.11 more than Jessica's initial deposit.