DATE: _____

Chain Rule Practice

1. Find $\left.\frac{dy}{dx}\right|_{x=\frac{\pi}{2}}$ given $y = \tan(\cos x)$

out: $\tan$ in: $\cos x$

$\frac{dy}{dx} = -\sin x \cdot \sec^2(\cos x)$

$\left.\frac{dy}{dx}\right|_{x=\pi} = -\sin\frac{\pi}{2} \cdot \sec^2\left(\cos\frac{\pi}{2}\right)$

$= -1 \cdot \sec^2(0)$

$= -1 \cdot \left(\frac{1}{\cos 0}\right)^2 = -1 \cdot \left(\frac{1}{1}\right)^2 = -1$

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2. If $y = 2\cos\frac{x}{2}$, then find $\frac{d^2y}{dx^2}$. → 2nd derivative

out: $2\cos$ in: $\frac{x}{2}$

$\frac{x}{2} = \frac{1}{2}x$ ... (1)

$\frac{dy}{dx} = \frac{1}{2} \cdot -2\sin\left(\frac{x}{2}\right)$

$= -\sin\left(\frac{x}{2}\right)$

out: $-\sin$ in: $\frac{x}{2}$

$\frac{d^2y}{dx^2} = \frac{1}{2} \cdot -\cos\left(\frac{x}{2}\right)$

$\boxed{\frac{d^2y}{dx^2} = -\frac{1}{2}\cos\left(\frac{x}{2}\right)}$

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3. Let the velocity of a particle be defined as $v(t) = \sin^2 \pi t$, where t is measured in seconds and $v(t)$ is measure in feet per second. Find the acceleration of the particle at $t = 2$.

→ $v'(t) = a(t)$ need $v'(2)$.

2 chain

$v(t) = \sin^2 \pi t$ ← out

$= (\sin(\pi t))^2$ in: πt out: $\sin$

$a(t) = v'(t) = \pi \cdot \cos(\pi t) \cdot 2(\sin \pi t)'$

$= 2\pi \cos \pi t \sin \pi t$

$a(2) = 2\pi \cos 2\pi \sin 2\pi$

$= 2\pi(1)(0)$

$\boxed{a(2) = 0}$

4. Find the slope of the line tangent to $f(x) = x(1-2x)^3$ at $(1,-1)$.

$f'(x)$ @ $(1,-1)$

in, out

f, g

product w/ chain

$f'(x) = (1-2x)^3 \cdot 1 + x \cdot -2 \cdot 2(1-2x)^2$

$= (1-2x)^3 - 4x(1-2x)^2$

$f'(1) = (1-2)^3 - 4(1)(1-2)^2$

$= -1 - 4(1)$

$= -1 - 4$

$\boxed{f'(1) = -5}$ slope of tangent line to f @ $(1,-1)$ is -5.

5. Find the equation of the tangent line to the graph of $f(x) = \sqrt{\sin x}$ at $x = \frac{\pi}{6}$.

$= (\sin x)^{1/2}$ ← out

m

$y - y_1 = m(x - x_1)$

$y - f(\frac{\pi}{6}) = f'(\frac{\pi}{6})(x - \frac{\pi}{6})$

$f(\frac{\pi}{6}) = (\sin\frac{\pi}{6})^{1/2}$

$= (\frac{1}{2})^{1/2} = \sqrt{\frac{1}{2}} = \frac{1}{\sqrt{2}}$

$\boxed{y - \frac{1}{\sqrt{2}} = \frac{\sqrt{6}}{4}(x - \frac{\pi}{6})}$

$f'(x) = \cos x \cdot \frac{1}{2}(\sin x)^{-1/2}$

$= \frac{\cos x}{2\sqrt{\sin x}}$

$f'(\frac{\pi}{6}) = \frac{\cos \pi/6}{2\sqrt{\sin \pi/6}}$

$= \frac{\frac{\sqrt{3}}{2}}{2\sqrt{\frac{1}{2}}}$

$= \frac{\frac{\sqrt{3}}{2}}{\frac{2}{\sqrt{2}}}$

$= \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2}$

$f'(\frac{\pi}{6}) = \frac{\sqrt{6}}{4}$