

Chain Rule Practice (including Polar, Vectors, and Parametric)

1. Find $\frac{dy}{dx}$ given $y = \tan(\cos x)$

$$\frac{dy}{dx} = (\sin x)(\sec^2(\cos x))$$

$$\frac{dy}{dx} = \sin x \sec^2(\cos x)$$

2. If $y = 2 \cos \frac{x}{2}$ then find $\frac{d^2y}{dx^2}$. 2nd derivative

$$\frac{dy}{dx} = \frac{1}{2} \cdot 2 \cos\left(\frac{x}{2}\right)$$

$$= \cos\left(\frac{x}{2}\right)$$

$$\frac{dy}{dx} = \underbrace{\cos}_{\text{out}}\left(\frac{x}{2}\right) \quad \underbrace{\frac{x}{2}}_{\text{inside}}$$

$$\frac{d^2y}{dx^2} = \frac{1}{2} \cos\left(\frac{x}{2}\right)$$

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3. Let the velocity vector be defined as $v(t) = \langle \sin^2 \pi t, \cos \pi t \rangle$, where t is measured in seconds and $v(t)$ is measured in feet. Find the acceleration vector at $t = 2$.

$$v(t) = \underbrace{\langle \sin(\pi t) \rangle}_{\text{inside}}^{\text{outer}}, \cos \pi t$$

$$\underbrace{\sin}_{\text{out}} \underbrace{\pi t}_{\text{in}}$$

$$a(t) = v'(t) = \langle \overbrace{\pi \cos(\pi t)}^{\text{der}}, \overbrace{2(\sin \pi t)}^{\text{der}} \rangle, \pi \cdot -\sin(\pi t)$$

$$= \langle 2\pi \cos \pi t \sin \pi t, -\pi \sin \pi t \rangle$$

$$\text{or} \quad \langle \pi \sin 2\pi t, -\pi \sin \pi t \rangle$$

$$a(2) = \langle \pi \sin 4\pi, -\pi \sin 2\pi \rangle = \boxed{\langle 0, 0 \rangle}$$

4. Find the slope of the line tangent to $f(x) = x(1-2x)^3$ at $(1, -1)$.

$$\underbrace{f \cdot g}_{\text{product}} \rightarrow gf' + fg'$$

$$f'(x) = (1-2x)^3(1) + x(-2 \cdot 3(1-2x)^2)$$

$$= (1-2x)^3 - 6x(1-2x)^2$$

$$\text{or} \quad = (1-2x)^2 [1-2x-6x]$$

$$= (1-2x)^2 (1-8x)$$

$$f'(1) = (-1)^2 (1-8)$$

$$\boxed{f'(1) = -7}$$

5. Given a curve defined by the parametric equation $x(t) = (2t^3 - 1)^4$ and $y(t) = \sqrt{\sin t}$. Find the slope of the line tangent to the curve in terms of t .

$$x'(t) = 6t^2 \cdot 4(2t^3 - 1)^3$$

$$y'(t) = \cos t \cdot \frac{1}{2}(\sin t)^{-\frac{1}{2}}$$

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{x'(t)}{y'(t)}$$

$$= \frac{\frac{1}{2} \cos t (\sin t)^{-\frac{1}{2}}}{24t^2(2t^3 - 1)^3}$$

$$\boxed{\frac{dy}{dx} = \frac{\cos t}{48t^2(2t^3 - 1)^3 \sqrt{\sin t}}}$$

6. Find the equation of the tangent line to the graph of $r = 3 - \cos 2\theta$ at $\theta = \pi$.

$$y - y_1 = m(x - x_1)$$

$$\begin{array}{c} y - 0 = \\ g - 0 = \\ (x - -2) \end{array}$$

$$x = r \cos \theta$$

$$x = (3 - \cos 2\theta) \cos \theta$$

$$\begin{aligned} x(\pi) &= (3 - \cos 2\pi) \cos \pi \\ &= (3 - 1)(-1) \\ &= -2 \end{aligned}$$

$$y = r \sin \theta$$

$$y = (3 - \cos 2\theta) \sin \theta$$

$$\begin{aligned} y(\pi) &= (3 - \cos 2\pi) \sin \pi \\ &= (3 - 1)(0) \\ &\approx 0 \end{aligned}$$

$$\frac{dy}{d\theta} = \frac{dy/d\theta}{dx/d\theta}$$

$$= \frac{\sin \theta (2 \sin 2\theta) + (3 - \cos 2\theta) \cos \theta}{\cos \theta (2 \sin 2\theta) + (3 - \cos 2\theta)(-\sin \theta)}$$

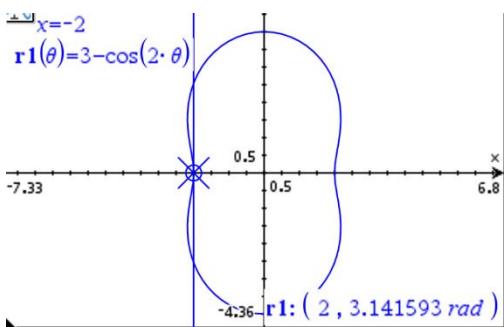
$$\left. \frac{dy}{dx} \right|_{\theta=\pi} = \frac{0(0) + (3 - 1)(-1)}{-1(0) + (3 - 1)(0)}$$

$$= \frac{-2}{0}$$

undefined / DNE

vertical lines have undefined slopes

$\therefore x = -2$ is the equation of the tangent line



Verified in graphing calculator