

3.6 Mathematics of Finance (Target 3G)

1. A man invests \$10,000 in an account that pays 8.5% interest per year, compounded quarterly. What is the amount of money that he will have after 3 years?

$$A = P \left(1 + \frac{r}{k}\right)^{kt}$$

$$A = 10000 \left(1 + \frac{.085}{4}\right)^{4(3)}$$

$$A = 12870.186 \rightarrow \boxed{\$12870.19}$$

$P = 10000$   
 $r = .085$   
 $k = 4$  (quarterly)  
 $t = 3$   
 $A = ?$

2. A sum of \$5000 is invested at an interest rate of 9% per year. Find the time required for the money to double if the interest is compounded:

(a) Semi-annually  $\rightarrow k=2$

$$10000 = 5000 \left(1 + \frac{.09}{2}\right)^{2t}$$

$$2 = (1.045)^{2t}$$

$$\ln 2 = \ln(1.045)^{2t}$$

$$\ln 2 = 2t \ln(1.045)$$

$$\frac{\ln 2}{2 \ln(1.045)} = t$$

$$\boxed{7.874 \text{ yrs} = t}$$

$P = 5000$   
 $r = 0.09$   
 $A = 10000$   
 $t = ?$

(b) Quarterly  $\rightarrow k=4$

$$10000 = 5000 \left(1 + \frac{.09}{4}\right)^{4t}$$

$$2 = (1.0225)^{4t}$$

$$\ln 2 = 4t \ln(1.0225)$$

$$\frac{\ln 2}{4 \ln(1.0225)} = t$$

$$\boxed{7.788 \text{ yrs} = t}$$

(c) Continuously

$$A = Pe^{rt}$$

$$10000 = 5000e^{.09t}$$

$$2 = e^{.09t}$$

$$\ln 2 = \ln e^{.09t}$$

$$\ln 2 = .09t \rightarrow t = \frac{\ln 2}{.09} = \boxed{7.702 \text{ yrs}}$$

3. How long will it take for \$8000 compounded monthly at 4% to grow to \$10000?

$$A = P \left(1 + \frac{r}{k}\right)^{kt}$$

$$10000 = 8000 \left(1 + \frac{.04}{12}\right)^{12t}$$

$$\frac{10000}{8000} = (1.0033)^{12t}$$

$$\ln\left(\frac{5}{4}\right) = 12t \ln(1.0033)$$

$$\boxed{t = 5.588 \text{ years}}$$

$A = 10000$   
 $P = 8000$   
 $k = 12$  (monthly)  
 $r = .04$   
 $t = ?$

4. How much money should you save in an account paying 5% interest compounded monthly if you want to have \$6000 in 6 months?

$$A = P \left(1 + \frac{r}{k}\right)^{kt}$$

$$6000 = P \left(1 + \frac{.05}{12}\right)^{12(.5)}$$

$$\frac{6000}{\left(1 + \frac{.05}{12}\right)^6} = P$$

$$5852.163 = P \rightarrow \boxed{\$5852.16}$$

$A = 6000$   
 $r = .05$   
 $k = 12$   
 $t = .5$  ( $\frac{1}{2}$  a year)  
 $P = ?$

5. A necklace is appraised at \$6300. If the value of the necklace has increased at an annual rate of 7%, how much was it worth 15 years ago?

$$A = P(1+r)^t$$

$$A = 6300(1+.07)^{-15}$$

$$A = 2283.410$$

$$\boxed{\$2283.41}$$

$A = 6300$   
 $r = .07$   
 $t = 15$   
 $P = ?$

$$A = P(1+r)^t$$

$$6300 = P(1+.07)^{15}$$

$$\frac{6300}{(1.07)^{15}} = P$$

$$\boxed{\$2283.41} = P$$