

Chain Rule & Polar Practice

In 1-4, derive each function.

1. $y = \underline{2x} \sin(3x)$ product

$$y' = \sin(3x) \cdot 2 + 2x \cdot \cos(3x) \cdot 3$$

$$y' = 2\sin 3x + 6x \cos 3x$$

2. $y = \overbrace{\tan(\cos x)}^{\text{out}} \quad \underbrace{\cos x}_{\text{in}}$

$$\frac{dy}{dx} = \sec^2(\cos x) \cdot -\sin x$$

$$\frac{dy}{dx} = -\sin x (\sec^2(\cos x))$$

3. $f(x) = \sqrt{3x^2 + 2x + 1}$

$$f(x) = (3x^2 + 2x + 1)^{\frac{1}{2}}$$

$$f'(x) = \frac{1}{2}(3x^2 + 2x + 1)^{-\frac{1}{2}} (6x + 2)$$

$$= (3x+1)(3x^2 + 2x + 1)^{-\frac{1}{2}}$$

$$f'(x) = \frac{3x+1}{\sqrt{3x^2 + 2x + 1}}$$

or

5. If $y = 2\cos\left(\frac{x}{2}\right)$, then find $\frac{d^2y}{dx^2}$.

$$\frac{dy}{dx} = -2 \sin\left(\frac{x}{2}\right) \cdot \frac{1}{2}$$

$$= -\sin\left(\frac{x}{2}\right)$$

$$\frac{d^2y}{dx^2} = -\cos\left(\frac{x}{2}\right) \cdot \frac{1}{2}$$

$$\frac{d^2y}{dx^2} = -\frac{1}{2} \cos\left(\frac{x}{2}\right)$$

4. $g(x) = \left(\frac{1-\cos x}{\sin x}\right)^3$

$$g'(x) = 3\left(\frac{1-\cos x}{\sin x}\right)^2 \left(\frac{\sin x(\sin x) - (1-\cos x)(\cos x)}{(\sin x)^2} \right)$$

$$= 3\left(\frac{1-\cos x}{\sin x}\right)^2 \left(\frac{\sin^2 x - \cos x + \cos^2 x}{\sin^2 x} \right)$$

$$= 3 \cdot \frac{(1-\cos x)^2}{\sin^2 x} \cdot \left(\frac{1-\cos x}{\sin^2 x} \right)$$

$$= 3 \cdot \frac{(1-\cos x)^2}{1-\cos^2 x} \cdot \frac{1-\cos x}{1-\cos^2 x}$$

$$= 3 \cdot \frac{(1-\cos x)(1-\cos x)(1-\cos x)}{(1-\cos x)(1+\cos x)(1-\cos x)(1+\cos x)}$$

$$= 3 \cdot \frac{1-\cos x}{(1+\cos x)^2}$$

$$g'(x) = \frac{3(1-\cos x)}{(1+\cos x)^2}$$

6. Find an equation of the line tangent to the graph of $f(x) = \underline{x(1-2x)^3}$ at the point $(1, -1)$.

$$y - y_1 = m(x - x_1)$$

$$\boxed{y + 1 = -7(x - 1)}$$

$$f'(x) = (1-2x)^3(1) + x(3(1-2x)^2 \cdot -2)$$

$$f'(x) = (1-2x)^3 - 6x(1-2x)^2$$

$$f'(1) = (1-2(1))^3 - 6(1)(1-2(1))^2$$

$$= -1 - 6(1)$$

$$= -1 - 6$$

$$= -7$$

7. Find the equation of the tangent line to the graph of $r = 3 - 2 \sin \theta$ at $\theta = \pi$.

$$y - y_1 = m(x - x_1)$$

$$y - 0 = \frac{3}{2}(x - 3)$$

$$\boxed{y = \frac{3}{2}(x+3)}$$

$$x = r \cos \theta$$

$$x = (3 - 2 \sin \theta) \cos \theta$$

$$\frac{dx}{d\theta} = (\cos \theta)(-2 \cos \theta) + (3 - 2 \sin \theta)(-\sin \theta)$$

$$y = r \sin \theta$$

$$y = (3 - 2 \sin \theta) \sin \theta$$

$$\frac{dy}{d\theta} = (\sin \theta)(-2 \cos \theta) + (3 - 2 \sin \theta)(\cos \theta)$$

$$\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{-2 \sin \theta \cos \theta + 3 \cos \theta - 2 \sin \theta \cos \theta}{-2 \cos^2 \theta - 3 \sin \theta - 2 \sin^2 \theta}$$

$$\frac{dy}{dx} \Big|_{\theta=\pi} = \frac{-2 \sin \pi \cos \pi + 3 \cos \pi - 2 \sin \pi \cos \pi}{-2(\cos \pi)^2 - 3 \sin \pi - 2(\sin \pi)^2}$$

$$= \frac{-3}{-2}$$

$$= \frac{3}{2}$$

$$\left. \begin{array}{l} x(\pi) = (3 - 2 \sin \pi) \cos \pi \\ \quad = -3 \\ y(\pi) = (3 - 2 \sin \pi) \sin \pi \\ \quad = 0 \end{array} \right\}$$

8. For $r = 3 - 2 \sin \theta$, find all points (r, θ) where the tangent line is horizontal.

$$\hookrightarrow \frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = \frac{-2 \sin \theta \cos \theta + 3 \cos \theta - 2 \sin \theta \cos \theta}{-2 \cos^2 \theta - 3 \sin \theta - 2 \sin^2 \theta}$$

$$0 = \frac{-4 \sin \theta \cos \theta + 3 \cos \theta}{-2(\cos^2 \theta + \sin^2 \theta) - 3 \sin \theta}$$

$$0 = -4 \sin \theta \cos \theta + 3 \cos \theta$$

$$0 = \cos \theta (-4 \sin \theta + 3)$$

$$\cos \theta = 0 \quad -4 \sin \theta + 3 = 0$$

$$\sin \theta = \frac{3}{4}$$

$$\theta = \frac{\pi}{2}, \frac{3\pi}{2}$$

$$\theta = .848, -.848$$

$$r = 3 - 2 \sin \frac{\pi}{2} = 3$$

$$r = 3 - 2 \sin (.848) = 3$$

$$r = 3 - 2 \sin \frac{3\pi}{2} = 5$$

$$r = 3 - 2 \sin (-.848) = 5$$

$$\boxed{(3, \frac{\pi}{2}) \quad (5, \frac{3\pi}{2}) \quad (3, .848) \quad (5, -.848)}$$