

Chain Rule Practice

In 1-4, derive each function.

1. $y = \underline{2x} \sin(\underline{3x})$ product

$$y' = \underline{\sin(3x)} \cdot \underline{2} + \underline{2x} \cdot \underline{3 \cdot \cos(3x)}$$

$$\boxed{y' = 2\sin 3x + 6x \cos 3x}$$

3. $f(x) = \sqrt{3x^2 + 2x + 1}$

$$= (\underline{3x^2 + 2x + 1})^{\frac{1}{2}}$$

$$f'(x) = (\underline{6x+2}) \cdot \frac{1}{2} (\underline{3x^2 + 2x + 1})^{-\frac{1}{2}}$$

$$\boxed{f'(x) = (3x+1)(3x^2 + 2x + 1)^{-\frac{1}{2}}}$$

5. Find an equation of the line tangent to the graph of $f(x) = \underline{x(1-2x)^3}$ at the point $(1, -1)$.

$$\begin{aligned} y - y_1 &= m(x - x_1) \\ y + 1 &= -7(x - 1) \end{aligned}$$

2. $y = \tan(\underline{\cos x})$

$$y' = -\sin x \cdot \sec^2(\cos x)$$

$$\boxed{y' = -\sin x \sec^2(\cos x)}$$

4. $g(x) = \left(\frac{1-\cos x}{\sin x}\right)^3$

$$g'(x) = \frac{(\sin x)(\sin x) - (1-\cos x)(\cos x)}{(\sin x)^2} \cdot 3 \left(\frac{1-\cos x}{\sin x}\right)^2$$

$$= \frac{\sin^2 x - \cos x + \cos^2 x}{(\sin x)^2} \cdot 3 \frac{(1-\cos x)^2}{(\sin x)^2}$$

$$= \frac{1 - \cos x}{\sin^2 x} \cdot 3 \frac{(1-\cos x)^2}{\sin^2 x}$$

$$= \frac{3(1-\cos x)^3}{(1-\cos^2 x)(1-\cos^2 x)}$$

$$= \frac{3(1-\cos x)^3}{(1-\cos x)(1+\cos x)(1-\cos x)(1+\cos x)}$$

$$= \frac{3(1-\cos x)^2}{(1+\cos x)^2}$$

$$= \boxed{\frac{3(1-\cos x)}{(1+\cos x)^2}}$$

$$f'(x) = (1-2x)^3 \cdot 1 + x \cdot -2 \cdot 3(1-2x)^2$$

$$= (1-2x)^3 - 6x(1-2x)^2$$

$$f'(1) = (1-2 \cdot 1)^3 - 6 \cdot 1(1-2 \cdot 1)^2$$

$$= (-1)^3 - 6(-1)^2$$

$$= -1 - 6$$

$$= -7$$

6. If $y = 2\cos\left(\frac{x}{2}\right)$, then find $\frac{d^2y}{dx^2}$. \leftarrow 2nd derivative

$$\begin{aligned} y' &= \frac{1}{2} \cdot -2\sin\left(\frac{x}{2}\right) \\ &= -\sin\left(\frac{x}{2}\right) \end{aligned}$$

$$\boxed{\frac{d^2y}{dx^2} = -\frac{1}{2} \cos\left(\frac{x}{2}\right)}$$

$$\frac{x}{2} = \frac{1}{2}x$$

