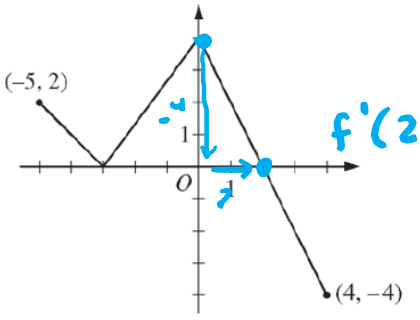
DATE: _____

AP FRQ: Chain Rule

No calculator is allowed for these problems.



Graph of f

1. The function f is defined on the closed interval $[-5,4]$. The graph of f consists of three line segments and is shown in the figure above. The function p is defined by $p(x) = f(x^2 - x)$. Find the slope of the line tangent to the graph of p at the point where $x = -1$.

$f'(2) = \frac{-4}{2} = -2$

↳ $p'(-1)$

$p'(x) = (2x-1)\, f'(x^2 - x)$

$p'(-1) = (2\cdot -1 - 1)\, f'((-1)^2 - -1)$

$= -3 f'(2)$

$p'(-1) = -3(-2)$

$\boxed{p'(-1) = 6}$

---

2. For $0 \le t \le 12$, a particle moves along the x-axis. The velocity of the particle at time t is given by $v(t) = \cos\left(\frac{\pi}{6}t\right)$. Find the acceleration of the particle at time t. Is the speed of the particle increasing, decreasing, or neither at time $t = 4$? Explain your reasoning.

$v(t), a(t)$ same signs — different signs?

$a(t) = -\frac{\pi}{6}\sin\left(\frac{\pi}{6}t\right)$

$v(4) = \cos\left(\frac{\pi}{6}\cdot 4\right)$

$= \cos\left(\frac{2\pi}{3}\right)$

$= -\frac{1}{2}$

$a(4) = -\frac{\pi}{6}\sin\left(\frac{\pi}{6}4\right)$

$= -\frac{\pi}{6}\cdot\frac{\sqrt{3}}{2}$

Speed inc @ $t=4$ b/c $v(4)<0$ and $a(4)<0$

---

3. At time t, a particle moving in the xy-plane is at position $(x(t), y(t))$, where $x(t)$ and $y(t)$ are not explicitly given. For $t \ge 0$, $\frac{dx}{dt} = 4t + 1$ and $\frac{dy}{dt} = \sin(t^2)$. At time $t = 0$, $x(0) = 0$ and $y(0) = -4$. Find the acceleration vector of the particle at time $t = 3$.

$a(t) = \langle 4, 2t \sin t^2 \rangle$

$a(3) = \langle 4, 2(3) \sin 3^2 \rangle$

$= \langle 4, 6\sin 9 \rangle$