DATE: _____

AP Calculus Free-Response Question
NON-Calculator

Consider the curve given by $xy^2 - x^3y = 6$.

a) Show that $\frac{dy}{dx} = \frac{3x^2y - y^2}{2xy - x^3}$

b) Find all points on the curve whose x-coordinate is 1, and write an equation for the tangent line at each of these points.

$y - y_1 = m(x - x_1)$

c) Find the x-coordinate of each point on the curve where the tangent line is vertical.

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a) $xy^2 - x^3y = 6$

$y^2 \cdot 1 + x \cdot 2y\frac{dy}{dx} - \left[y \cdot 3x^2 + x^3\frac{dy}{dx}\right] = 0$

$y^2 + 2xy\frac{dy}{dx} - 3x^2y - x^3\frac{dy}{dx} = 0$

$2xy\frac{dy}{dx} - x^3\frac{dy}{dx} = 3x^2y - y^2$

$\frac{dy}{dx}(2xy - x^3) = 3x^2y - y^2$

$\boxed{\frac{dy}{dx} = \frac{3x^2y - y^2}{2xy - x^3}}$

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b) $xy^2 - x^3y = 6$

$1y^2 - 1^3y = 6$

$y^2 - y = 6$

$y^2 - y - 6 = 0$

$(y-3)(y+2) = 0$

$y = 3, \; y = -2$

$\boxed{(1,3) \text{ and } (1,-2)}$

$\left.\frac{dy}{dx}\right|_{(1,3)} = \frac{3(1)^2(3) - 3^2}{2(1)(3) - 1^3}$

$= \frac{9-9}{6-1}$

$= 0$

$\boxed{y - 3 = 0(x-1)}$

$\left.\frac{dy}{dx}\right|_{(1,-2)} = \frac{3(1)^2(-2) - (-2)^2}{2(1)(-2) - 1^3}$

$= \frac{-6-4}{-4-1}$

$= \frac{-10}{-5}$

$= 2$

$\boxed{y + 2 = 2(x-1)}$

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c) $\frac{dy}{dx} = \frac{3x^2y - y^2}{2xy - x^3}$

$\frac{1}{0} = \frac{3x^2y - y^2}{2xy - x^3}$

$2xy - x^3 = 0$

$x(2y - x^2) = 0$

~~$x = 0$~~ $\quad 2y - x^2 = 0$

$2y = x^2$

$y = \frac{x^2}{2}$

$xy^2 - x^3y = 6$

$0 \cdot y^2 - 0^3y = 6$

$0 \neq 6$

not true...
so $x = 0$
doesn't work

$xy^2 - x^3y = 6$

$x\left(\frac{x^2}{2}\right)^2 - x^3\left(\frac{x^2}{2}\right) = 6$

$\frac{x^5}{4} - \frac{x^5}{2} = 6$

$\frac{x^5}{4} - \frac{2x^5}{4} = 6$

$-\frac{x^5}{4} = 6$

$\sqrt[5]{x^5} = \sqrt[5]{-24}$

$\boxed{x = \sqrt[5]{-24}}$