

Derivatives of Inverse Trig Functions Practice

1. Evaluate $\lim_{h \rightarrow 0} \frac{\arccos\left(\frac{1}{2}+h\right) - \arccos\left(\frac{1}{2}\right)}{h}$.

definition of derivative

where $f(x) = \arccos(x)$ and $x = \frac{1}{2}$

$$f'(x) = -\frac{1}{\sqrt{1-x^2}}$$

$$f'\left(\frac{1}{2}\right) = -\frac{1}{\sqrt{1-\left(\frac{1}{2}\right)^2}} = -\frac{1}{\sqrt{1-\frac{1}{4}}} = -\frac{1}{\sqrt{\frac{3}{4}}} = -\frac{1}{\frac{\sqrt{3}}{2}} = \boxed{-\frac{2}{\sqrt{3}}}$$

2. The position of a particle moving in the xy -plane is given by the parametric equations

$x(t) = \sin^{-1}(4t^3 - 3t^2)$ and $y(t) = \tan^{-1}(t\sqrt{t-1})$. For what value of t is the particle at rest?

$$\begin{aligned} x'(t) &= (12t^2 - 6t) \cdot \frac{1}{\sqrt{1-(4t^3-3t^2)^2}} \\ &= \frac{12t^2 - 6t}{\sqrt{1-t^4 + 8t^3 - 12t^2}} \end{aligned}$$

$x'(t) = 0$

$$0 = 12t^2 - 6t$$

$$0 = 6t(2t-1)$$

$$t=0, t=\frac{1}{2}$$

$$\begin{aligned} y'(t) &= (\sqrt{t-1})(1) + (t)(\frac{1}{2}(t-1)^{-\frac{1}{2}}) \cdot \frac{1}{1+t\sqrt{t-1}} \text{ and } y'(t)=0 \\ &= \left(\sqrt{t-1} + \frac{t}{\sqrt{t-1}}\right) \left(\frac{1}{1+t\sqrt{t-1}}\right) \\ &= \frac{t-1+t}{\sqrt{t-1}} \cdot \frac{1}{1+t^3-t^2} \\ &= \frac{2t-1}{(\sqrt{t-1})(1+t^3-t^2)} \\ &0 = 2t-1 \\ &\frac{1}{2} = t \end{aligned}$$

Particle is resting when $t = \frac{1}{2}$ b/c $x'(\frac{1}{2}) = 0$ and $y'(\frac{1}{2}) = 0$

3. Determine where $f(x) = 2x + 10 \arccot x$ has a horizontal tangent line.

$\hookrightarrow f' = 0$

$$f'(x) = 2 + 10\left(-\frac{1}{1+x^2}\right)$$

$$0 = 2 - \frac{10}{1+x^2}$$

$$\frac{10}{1+x^2} = 2$$

$$10 = 2 + 2x^2$$

$$8 = 2x^2$$

$$4 = x^2$$

$$\pm 2 = x$$

$f(x)$ has a horizontal tangent line

when $x = 2$ or $x = -2$

4. Find $\frac{dy}{dx}$ for $\tan^{-1}(x^2y) = x + 3xy^2$

$$\left(\underbrace{y^2 \cdot 2x + x^2 \frac{dy}{dx}}_{\text{d in}} \right) \cdot \underbrace{\frac{1}{1+(x^2y)^2}}_{\text{d out}} = 1 + 3 \left(\underbrace{y^2 \cdot 1 + x \cdot 2y \frac{dy}{dx}}_{\text{product}} \right)$$

$$(2xy + x^2 \frac{dy}{dx}) \cdot \frac{1}{1+x^4y^2} = 1 + 3y^2 + 6xy \frac{dy}{dx}$$

$$(1+x^4y^2) \cdot \frac{2xy + x^2 \frac{dy}{dx}}{1+x^4y^2} = (1+3y^2+6xy \frac{dy}{dx})(1+x^4y^2)$$

$$2xy + x^2 \frac{dy}{dx} = 1 + x^4y^2 + 3y^2 + 3x^4y^4 + 6xy(1+x^4y^2) \frac{dy}{dx}$$

$$x^2 \frac{dy}{dx} - 6xy(1+x^4y^2) \frac{dy}{dx} = 1 + x^4y^2 + 3y^2 + 3x^4y^4 - 2xy$$

$$\frac{dy}{dx} (x^2 - 6xy) = 1 + x^4y^2 + 3y^2 + 3x^4y^4 - 2xy$$

$$\boxed{\frac{dy}{dx} = \frac{1+x^4y^2+3y^2+3x^4y^4-2xy}{x^2-6xy}}$$