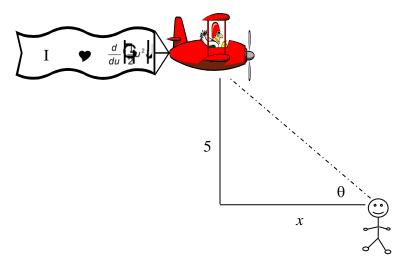
Derivatives of Inverse Trig Functions

You are looking up at a plane flying about 5 miles above the ground. As the plane move closer to you, the angle of your head/eyes changes. What is the rate at which that angle is changing with respect to x?



Derivatives of Inverse Trig Functions

$$\frac{d}{dx}(\sin^{-1}x) = \frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}(\cos^{-1}x) = -\frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}(\tan^{-1}x) = \frac{1}{1+x^2}$$

$$\frac{d}{dx}(\cot^{-1}x) = -\frac{1}{1+x^2}$$

$$\frac{d}{dx}(\sec^{-1}x) = \frac{1}{|x|\sqrt{x^2-1}}$$

$$\frac{d}{dx}(\csc^{-1}x) = -\frac{1}{|x|\sqrt{x^2-1}}$$

Example 1:

Find
$$\frac{dy}{dx}$$
 for $y = x \sin^{-1}(x)$

Example 2:

Given position of an object as described by $x(t) = \frac{\tan^{-1} t}{t^2 + 3}$ where $t \ge 0$. Find the velocity of the object when t = 1.

Derivatives of Inverse Trig Functions

$$\frac{d}{dx}(\sin^{-1} f(x)) = f'(x) \cdot \frac{1}{\sqrt{1 - (f(x))^2}}$$

$$\frac{d}{dx}(\cos^{-1}f(x)) = -f'(x) \cdot \frac{1}{\sqrt{1 - (f(x))^2}}$$

$$\frac{d}{dx}(\tan^{-1} f(x)) = f'(x) \cdot \frac{1}{1 + (f(x))^2}$$

$$\frac{d}{dx}(\cot^{-1}f(x)) = -f'(x) \cdot \frac{1}{1 + (f(x))^2}$$

$$\frac{d}{dx}(\sec^{-1}x) = f'(x) \cdot \frac{1}{|f(x)|\sqrt{(f(x))^2 - 1}} \qquad \frac{d}{dx}(\csc^{-1}x) = -f'(x) \cdot \frac{1}{|f(x)|\sqrt{(f(x))^2 - 1}}$$

$$\frac{d}{dx}(\csc^{-1}x) = -f'(x) \cdot \frac{1}{|f(x)|\sqrt{(f(x))^2 - 1}}$$

Example 3:

Find
$$g'(x)$$
 for $g(x) = \cot^{-1} \sqrt{x}$

Example 4:

Write the equation of the line tangent to the curve $h(x) = \cos^{-1}\left(\frac{x}{2} - 1\right)$ at x = 3.