DATE: _____

1. Write the equation(s) of the tangent line(s) for $x^2 - y^2 = 3x$ at $x = -1$.

$2x - 2y\frac{dy}{dx} = 3$

$-2y\frac{dy}{dx} = 3 - 2x$

$\frac{dy}{dx} = \frac{3-2x}{-2y}$

$(-1)^2 - y^2 = (3)(-1)$

$1 - y^2 = -3$

$-y^2 = -4$

$y^2 = 4$

$y = \pm 2$

$\left.\frac{dy}{dx}\right|_{(-1,2)} = \frac{3-2(-1)}{-2(2)}$

$= \frac{5}{-4}$

$y - 2 = -\frac{5}{4}(x+1)$

$\left.\frac{dy}{dx}\right|_{(-1,-2)} = \frac{3-2(-1)}{-2(-2)}$

$= \frac{5}{4}$

$y + 2 = \frac{5}{4}(x+1)$

2. Prove that the slope of the tangent line at $(1,2)$ is -3 for $x^2 - 4xy + y = 3x - 8$.

$2x - 4\left(y + x\frac{dy}{dx}\right) + \frac{dy}{dx} = 3$

$2x - 4y - 4x\frac{dy}{dx} + \frac{dy}{dx} = 3$

$2(1) - 4(2) - 4(1)\frac{dy}{dx} + \frac{dy}{dx} = 3$

$2 - 8 - 4\frac{dy}{dx} + \frac{dy}{dx} = 3$

$-6 - 3\frac{dy}{dx} = 3$

$-3\frac{dy}{dx} = 9 \longrightarrow \left.\frac{dy}{dx}\right|_{(1,2)} = -3$

3. Find the value of $\left.\frac{d^2y}{dx^2}\right|_{(1,2)}$ for the given equation $x^2 - 4xy + y = 3x - 8$.

$\frac{dy}{dx} = \frac{3 - 2x + 4y}{1 - 4x}$ (from part 2) ☺

$\frac{d^2y}{dx^2} = \frac{(1-4x)\left(-2 + 4\frac{dy}{dx}\right) - (3 - 2x + 4y)(-4)}{(1-4x)^2}$

$\left.\frac{d^2y}{dx^2}\right|_{(1,2)} = \frac{(1-4\cdot1)(-2 + 4\cdot -3) - (3 - 2\cdot1 + 4\cdot2)(-4)}{(1-4\cdot1)^2}$

$= \frac{-3(-14) - (9)(-4)}{(-3)^2}$

$= \frac{42 + 36}{9}$

$= \frac{78}{9} = \boxed{\frac{26}{3}}$