

Derivatives of Inverse Functions

Recall: Finding the Inverse of a Function

- ① switch x & y
- ② solve for y
- ③ write with $f^{-1}(x)$ notation

Example: $f(x) = x^3$

$$x = y^3$$

$$\sqrt[3]{x} = y$$

$$f^{-1}(x) = \sqrt[3]{x}$$

Example: Find $(f^{-1})'(8)$ where $f(x) = x^3$.

From above

$$f^{-1}(x) = \sqrt[3]{x}$$

$$(f^{-1})'(x) = \frac{1}{3}x^{-2/3}$$

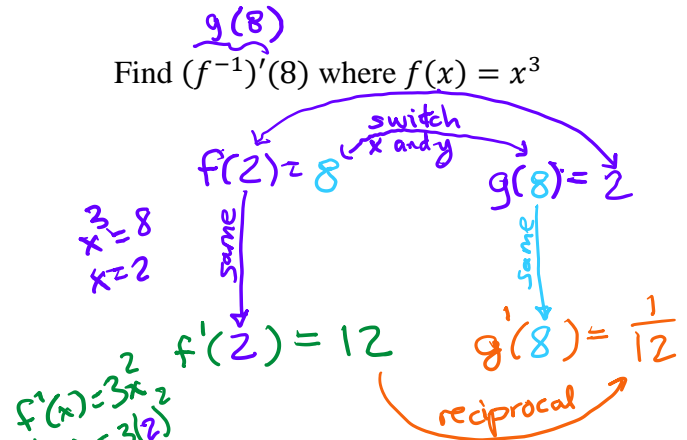
$$(f^{-1})'(8) = \frac{1}{3}(8)^{-2/3} = \frac{1}{3} \cdot \left(\frac{1}{\sqrt[3]{8}}\right)^2 = \frac{1}{3} \cdot \frac{1}{4} = \boxed{\frac{1}{12}}$$

Derivative of Inverse Functions

Find $(f^{-1})'(8)$ where $f(x) = x^3$

- ① get x & y -values for $f(x)$ and $f^{-1}(x)$

let $g(x) = f^{-1}(x)$



- ② find $f'(x)$ at f 's x -value

$\frac{\Delta y}{\Delta x}$

- ③ find $(f^{-1})'(x)$ at f^{-1} 's x -value

$\frac{\Delta x}{\Delta y}$

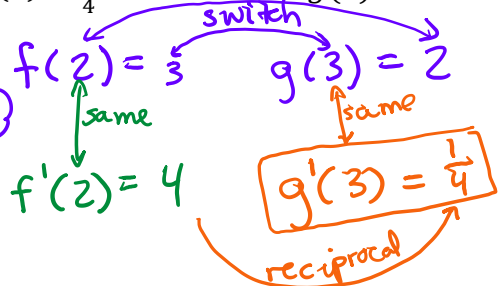
$g'(x)$ g

$f' \rightarrow \frac{\Delta y}{\Delta x}$ $(f^{-1})' \rightarrow \frac{\Delta x}{\Delta y}$

Example 1:

Given $f(x) = \frac{1}{4}x^3 + x - 1$ and $g(x)$ is the inverse of $f(x)$, find $g'(3)$.

insolve $\frac{1}{4}x^3 + x - 1 = 3$
 $\frac{d}{dx}(\frac{1}{4}x^3 + x - 1) = 2$



Example 2:

Given $f(x) = x^2 + 3x - 1$ for $x \geq -1$, find $(f^{-1})'(3)$.

$$\begin{aligned}x^2 + 3x - 1 &= 3 \\x^2 + 3x - 4 &= 0 \\(x+4)(x-1) &= 0 \\x &= -4, x = 1 \\x &\geq -1\end{aligned}$$

$$f(1) = 3 \quad (f^{-1})(3) = 1$$

$$f'(1) = 5$$

$$(f^{-1})'(3) = \frac{1}{5}$$

reciprocal

$$\begin{aligned}f'(x) &= 2x + 3 \\f'(1) &= 2(1) + 3\end{aligned}$$

Example 3:

The table below gives values of a function f and its derivative f' .
If f has the inverse function f^{-1} , find $(f^{-1})'(2)$.

let $g = f^{-1}$

x	$f(x)$	$f'(x)$
-1	2	-6
1	-1	-4
2	-3	-7
8	-5	-1

$$f(-1) = 2$$

$$g(2) = -1$$

$$f'(-1) = -6$$

$$g'(2) = -\frac{1}{6}$$