DATE:_____

Derivatives of Inverse Functions

Recall: Finding the Inverse of a Function

① switch x & y

② solve for y

③ write with $f^{-1}(x)$ notation

Example: $f(x) = x^3$

$x = y^3$

$\sqrt[3]{x} = y$

$f^{-1}(x) = \sqrt[3]{x}$

Example: Find $(f^{-1})'(8)$where $f(x) = x^3$.

from above

$f^{-1}(x) = \sqrt[3]{x}$

$(f^{-1})'(x) = \frac{1}{3}x^{-2/3}$

$(f^{-1})'(8) = \frac{1}{3}(8)^{-2/3} = \frac{1}{3}\cdot\left(\frac{1}{\sqrt[3]{8}}\right)^2 = \frac{1}{3}\cdot\frac{1}{4} = \boxed{\frac{1}{12}}$

Derivative of Inverse Functions

Find $(f^{-1})'(8)$ where $f(x) = x^3$

$g(8)$

① get x & y-values for $f(x)$ and $f^{-1}(x)$

let $g(x) = f^{-1}(x)$

$x^3 = 8$

$x = 2$

$f(2) = 8$ (switch x and y) $g(8) = 2$

② find $f'(x)$ at f's x-value

$\frac{\Delta y}{\Delta x}$

$f'(x) = 3x^2$

$f'(2) = 3(2)^2$

same

same

$f'(2) = 12$

$g'(8) = \frac{1}{12}$

reciprocal

③ find $(f^{-1})'(x)$ at f^{-1}'s x-value

$\frac{\Delta x}{\Delta y}$

$g'(x)$ g

$f' \to \frac{\Delta y}{\Delta x}$ $(f^{-1})' \to \frac{\Delta x}{\Delta y}$

Example 1:

Given $f(x) = \frac{1}{4}x^3 + x - 1$ and $g(x)$ is the inverse of $f(x)$, find $g'(3)$.

switch

$f(2) = 3$ $g(3) = 2$

same

same

$f'(2) = 4$

$g'(3) = \frac{1}{4}$

reciprocal

③ solve $\frac{1}{4}x^3 + x - 1 = 3$

④ $f'(x)|_{x=2}$

Example 2:

Given $f(x) = x^2 + 3x - 1$ for $x \geq -1$, find $(f^{-1})'(3)$.

$x^2+3x-1=3$
$x^2+3x-4=0$
$(x+4)(x-1)=0$
$x=-4$ not ≥ -1, $x=1$

$f(1)=3 \quad (f^{-1})(3)=1$

$f'(x)=2x+3$
$f'(1)=2(1)+3$

$f'(1)=5$

$(f^{-1})'(3)=\frac{1}{5}$

reciprocal

Example 3:

The table below gives values of a function f and its derivative f'.
If f has the inverse function f^{-1}, find $(f^{-1})'(2)$.

let $g=f^{-1}$

x	$f(x)$	$f'(x)$
-1	2	-6
1	-1	-4
2	-3	-7
8	-5	-1

$f(-1)=2 \quad g(2)=-1$

$f'(-1)=-6$

$g'(2)=-\frac{1}{6}$