

## Derivatives of Inverse Functions

1. The function  $f(x) = x^5 + 3x - 2$  passes through the point  $(1, 2)$ . Let  $f^{-1}$  denote the inverse of  $f$ . Then  $(f^{-1})'(2)$  equals:

(A)  $\frac{1}{83}$

(B)  $\frac{1}{8}$

(C) 1

(D) 8

(E) 83

$$\begin{aligned} f(x) &= x^5 + 3x - 2 \\ f'(x) &= 5x^4 + 3 \\ f'(1) &= 5(1)^4 + 3 \end{aligned}$$

$$\begin{aligned} f(1) &= 2 & \text{switch} \\ f'(1) &= 8 & (f^{-1})(2) = 1 \\ & \uparrow & (f^{-1})'(2) = \frac{1}{8} \\ & \text{reciprocal} & \end{aligned}$$

2. If  $f(2) = 3$ ,  $f'(2) = 4$ , and  $g(x)$  is the inverse function to  $f(x)$ , the equation of the tangent line to  $g(x)$  at  $x = 3$  is:

(A)  $y - 2 = -\frac{1}{4}x - 3$

(B)  $y - 2 = 4(x - 3)$

(C)  $y - 3 = -\frac{1}{4}x - 2$

(D)  $y - 2 = \frac{1}{4}(x - 3) \rightarrow y - y_1 = m(x - x_1)$

(E)  $y - 3 = \frac{1}{4}(x - 2)$

pt. for  $g(x) \rightarrow (3, 2)$ 

$$\begin{aligned} f(2) &= 3 & \text{switch} \\ f'(2) &= 4 & g(3) = 2 \\ & \uparrow & \uparrow \\ & \text{reciprocal} & \end{aligned}$$

3. If  $f(x) = 3x^2 - x$ , and  $g(x) = f^{-1}(x)$ , then  $g'(10)$  could be

(A) 59      (B)  $\frac{1}{59}$

(C)  $\frac{1}{10}$

(D) 11

(E)  $\frac{1}{11}$

$$\begin{aligned} f(x) &= 10 \\ 3x^2 - x - 10 &= 0 \\ 3x^2 - x - 10 &= 0 \\ (3x+5)(x-2) &= 0 \\ x &= -\frac{5}{3}, 2 \\ f'(x) &= 6x - 1 \\ f'(2) &= 11 \end{aligned}$$

$$\begin{aligned} f(2) &= 10 & \text{switch} \\ f'(2) &= 11 & g'(10) = 2 \\ & \uparrow & \uparrow \\ & \text{reciprocal} & \end{aligned}$$

4. Let  $f$  be a differentiable function such that  $f(3) = 15$ ,  $f(6) = 3$ ,  $f'(3) = -8$ , and  $f'(6) = -2$ . The function  $g$  is differentiable and  $g(x) = f^{-1}(x)$  for all  $x$ . What is the value of  $g'(3)$ ?

(A)  $-\frac{1}{2}$

(B)  $-\frac{1}{8}$

(C)  $\frac{1}{6}$

(D)  $\frac{1}{3}$

$f(6) = 3$

$g(3) =$

$f'(6) = -2$

$g'(3) = -\frac{1}{2}$

 $\text{reciprocal}$