DATE: ____________

Derivatives of Exponential Functions (polar, parametric, vector)

1. Given the position vector of a particle as $\langle \cos^{-1} t, 8t - 10e^{2t-1} \rangle$, determine what direction the particle is moving when $t = \frac{1}{2}$.

$v(t) = \langle -\frac{1}{\sqrt{1-t^2}}, 8 - 10 \cdot 2e^{2t-1} \rangle$

$v(\frac{1}{2}) = \langle -\frac{1}{\sqrt{1-(\frac{1}{2})^2}}, 8 - 20e^{2(\frac{1}{2})-1} \rangle$

$= \langle -\frac{1}{\sqrt{1-1/4}}, 8 - 20e^0 \rangle$

$= \langle -\frac{2}{\sqrt{3}}, -12 \rangle$

The particle is moving left and down @ $t = \frac{1}{2}$ b/c $x'(\frac{1}{2}) < 0$ and $y'(\frac{1}{2}) < 0$

2. Find the tangent line for the polar curve $r = e^{-\theta}$ at $\theta = 0$.

$y - y_1 = m(x - x_1)$

$\boxed{y - 0 = -1(x - 1)}$

$x = e^{-\theta}\cos\theta$ $\quad$ $y = e^{-\theta}\sin\theta$

$x(0) = e^0 \cos 0 = 1$ $\quad$ $y(0) = e^0 \sin 0 = 0$

$\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} = \frac{\sin\theta(-e^{-\theta}) + e^{-\theta}\cos\theta}{\cos\theta(-e^{-\theta}) + e^{-\theta}(-\sin\theta)} = \frac{e^{-\theta}(-\sin\theta + \cos\theta)}{e^{-\theta}(-\cos\theta - \sin\theta)} = \frac{-\sin\theta + \cos\theta}{-\cos\theta - \sin\theta}$

$\left.\frac{dy}{dx}\right|_{\theta=0} = \frac{-\sin 0 + \cos 0}{-\cos 0 - \sin 0}$

$= \frac{1}{-1}$

$= -1$

3. A particle is moving along a curve so that its position at time t is defined by $x(t) = t^2 e^{-3t}$ and $y(t) = -1 + e^{\cos t}$. Determine at what time t, where $0 \le t \le 2\pi$, the particle is resting, if there is a time.

$x'(t) = e^{-3t}(2t) + t^2(-3e^{-3t})$

$0 = te^{-3t}(2-3t)$

$t = 0$, $\quad e^{-3t} = 0$ $\quad 2 - 3t = 0$

$\quad$ never happens $\quad t = 2/3$

$y'(t) = -\sin t\, e^{\cos t}$

$0 = -\sin t\, e^{\cos t}$

$-\sin t = 0$ $\quad e^{\cos t} = 0$

$t = 0, \pi, 2\pi$ $\quad$ never happens

The particle is resting @ $t = 0$ b/c $x'(0) = 0$ and $y'(0) = 0$.