

Derivatives of Exponential Functions

$$\frac{d}{dx}(e^x) = e^x$$

Example 1:

- Given $f(x) = 2e^x$, find $f'(3)$.

$$\begin{aligned} f'(x) &= 2e^x \\ f'(3) &= 2e^3 \end{aligned}$$

Example 2:

- Given $g(x) = e^{2x}$, find $g'(x)$.

$$g'(x) = 2e^{2x}$$

Example 3:

- Given $h(x) = \frac{e^{3x+1}}{x^3+4}$, find $h'(x)$.

$$h'(x) = \frac{(x^3+4)(3e^{3x+1}) - (e^{3x+1})(3x^2)}{(x^3+4)^2}$$

$$= \frac{3e^{3x+1}(x^3+4 - 3x^2)}{(x^3+4)^2} = \boxed{\frac{3e^{3x+1}(x^3 - x^2 + 4)}{(x^3+4)^2}}$$

Example 4:

- The table below gives values of a function f and its derivative f' .

- If $p(x) = f(x)(e^{x-1} - 2x)$, find the equation of the line tangent to $p(x)$ at $x = 1$.

$$\begin{aligned} p(1) &= f(1) \cdot e^{1-1} - 2f(1) \\ &= 4(e^0 - 2) \\ &= 4(1 - 2) \\ &= -4 \end{aligned}$$

x	$f(x)$	$f'(x)$
-1	0	-6
0	-1	-4
1	4	7

$$\begin{aligned} y - y_1 &= m(x - x_1) \\ y - p(1) &= p'(1)(x - 1) \\ y + 4 &= -11(x - 1) \end{aligned}$$

$$p'(x) = (e^{x-1} - 2x)f'(x) + f(x)[1 \cdot e^{x-1} - 2]$$

$$\begin{aligned} p'(1) &= (e^0 - 2)f'(1) + f(1)(e^0 - 2) \\ &= (-1)7 + 4(-1) = -11 \end{aligned}$$

Example 5:

- Find y'' , where $y = \sqrt[3]{e^{x+1}}$.

2 chains?

$$= (e^{x+1})^{\frac{1}{3}}$$

$$= e^{\frac{1}{3}x + \frac{1}{3}}$$

$$y' = \frac{1}{3}e^{\frac{1}{3}x + \frac{1}{3}}$$

$$y'' = \frac{1}{3} \cdot \frac{1}{3}e^{\frac{1}{3}x + \frac{1}{3}}$$

$$y'' = \frac{1}{9}e^{\frac{1}{3}x + \frac{1}{3}}$$

only one chain!

Example 6:

- Find $\frac{dy}{dx}$ in terms of x for $e^x + e^y = x^3$.

implicit

$$e^x + e^y \frac{dy}{dx} = 3x^2$$

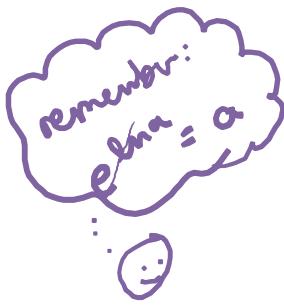
$$e^y \frac{dy}{dx} = 3x^2 - e^x$$

$$\frac{dy}{dx} = \frac{3x^2 - e^x}{e^y}$$

$$\boxed{\frac{dy}{dx} = \frac{3x^2 - e^x}{x^3 - e^x}}$$

$$e^x + e^y = x^3$$

$$e^y = x^3 - e^x$$



$$\frac{d}{dx}(a^x) = \ln a \cdot a^x$$



$$\begin{aligned}\frac{d}{dx}(a^x) &= \cancel{a} \cdot (e^{\ln a})^x \\ &= \ln a \cdot e^{\ln a \cdot x} \\ &= \ln a \cdot a^x\end{aligned}$$

Example 1:

Given $f(x) = 4^{3x^2-2}$, find the slope of $f(x)$ at $x = 1$.
↳ $f'(1) ?$

$$f'(x) = \ln 4 \cdot 6x \cdot 4^{3x^2-2}$$

$$f'(1) = \ln 4 \cdot 6 \cdot 4$$

$$= \boxed{24 \ln 4} \text{ or } 24 \ln 2^3 = 2 \cdot 24 \ln 2 \\ = 48 \ln 2$$

Example 2:

Given $g(x) = x \cdot 5^{2x}$, find $g'(x)$.

$$\begin{aligned}g'(x) &= 5^{2x} \cdot 1 + x \cdot \ln 5 \cdot 2 \cdot 5^{2x} \\ &= 5^{2x} + 2x \ln 5 \cdot 5^{2x}\end{aligned}$$

$$\alpha = 5^{2x} (1 + x \cdot 2 \ln 5)$$