DATE:_____

Derivatives of Exponential Functions

$$\frac{d}{dx}(e^x) = e^x$$

Example 1:

Given $f(x) = 2e^x$, find $f'(3)$.

$f'(x) = 2e^x$

$f'(3) = 2e^3$

Example 2: out ... in

Given $g(x) = e^{2x}$, find $g'(x)$.

$g'(x) = 2e^{2x}$

Example 3: out ... in — quotient

Given $h(x) = \frac{e^{3x+1}}{x^3+4}$, find $h'(x)$.

$$h'(x) = \frac{(x^3+4)(3e^{3x+1}) - (e^{3x+1})(3x^2)}{(x^3+4)^2}$$

$$= \frac{3e^{3x+1}(x^3+4-x^2)}{(x^3+4)^2} = \frac{3e^{3x+1}(x^3-x^2+4)}{(x^3+4)^2}$$

Example 4:

The table below gives values of a function f and its derivative f'.

If $p(x) = f(x)(e^{x-1} - 2x)$, find the equation of the line tangent to $p(x)$ at $x = 1$.

x	$f(x)$	$f'(x)$
-1	0	-6
0	-1	-4
1	4	7

$p(1) = f(1)(e^{1-1} - 2(1))$

$= 4(e^0 - 2)$

$= 4(1-2)$

$= -4$

$y - y_1 = m(x - x_1)$

$y - p(1) = p'(1)(x-1)$

$y + 4 = -11(x-1)$

$p'(x) = (e^{x-1} - 2x)f'(x) + f(x)[1 \cdot e^{x-1} - 2]$

$p'(1) = (e^0 - 2)f'(1) + f(1)(e^0 - 2)$

$= (-1)7 + 4(-1) = -11$

Example 5:

Find y'', where $y = \sqrt[3]{e^{x+1}}$.

2 chains?

$= (e^{x+1})^{1/3}$

$= e^{\frac{1}{3}x + \frac{1}{3}}$

only one chain!

$y' = \frac{1}{3} e^{\frac{1}{3}x+\frac{1}{3}}$

$y'' = \frac{1}{3} \cdot \frac{1}{3} e^{\frac{1}{3}x+\frac{1}{3}}$

$y'' = \frac{1}{9} e^{\frac{1}{3}x+\frac{1}{3}}$

Example 6:

Find $\frac{dy}{dx}$ in terms of x for $e^x + e^y = x^3$.

implicit

$e^x + e^y \frac{dy}{dx} = 3x^2$

$e^y \frac{dy}{dx} = 3x^2 - e^x$

$\frac{dy}{dx} = \frac{3x^2 - e^x}{e^y}$

$e^x + e^y = x^3$

$e^y = x^3 - e^x$

$\frac{dy}{dx} = \frac{3x^2 - e^x}{x^3 - e^x}$

remember: $e^{\ln a} = a$

$$\frac{d}{dx}(a^x) = \ln a \cdot a^x$$

$\frac{d}{dx}(a^x) = \frac{d}{dx}(e^{\ln a^x})$
$= \ln a \cdot e^{\ln a^x}$
$= \ln a \cdot a^x$

Example 1:

Given $f(x) = 4^{(3x^2-2)}$, find the slope of $f(x)$ at $x = 1$. → $f'(1)$?

$f'(x) = \ln 4 \cdot 6x \cdot 4^{3x^2-2}$

$f'(1) = \ln 4 \cdot 6 \cdot 4$

$= \boxed{24 \ln 4}$ or $24 \ln 2^2 = 2 \cdot 24 \ln 2$
$= 48 \ln 2$

Example 2:

Given $g(x) = x \cdot 5^{2x}$, find $g'(x)$.

$g'(x) = 5^{2x} \cdot 1 + x \cdot \ln 5 \cdot 2 \cdot 5^{2x}$

$= 5^{2x} + 2x \ln 5 \cdot 5^{2x}$

or $= 5^{2x}(1 + x 2 \ln 5)$