Derivatives of Logarithmic Functions

1. Suppose $f(1)=2$, $f'(1)=3$, and $f'(2)=4$. Then $\left(f^{-1}\right)'(2)=?$

A. $-\frac{1}{3}$

B. $-\frac{1}{4}$

C. $\frac{1}{4}$

D. $\frac{1}{3}$

E. cannot be determined

$f(1)=2 \qquad (f^{-1})(2)$

$f'(1)=3 \qquad (f^{-1})'(2)=\frac{1}{3}$

2. Suppose $y=f(x)=2x^3-3x$. If $h(x)$ is the inverse function of *f*, then $h'(-1)$ = ?

A. -1

B. $\frac{1}{5}$

C. $\frac{1}{3}$

D. 1

E. 3

$f'(x)=6x^2-3$

$2(1)^3-3(1)=1$

$f(1)=-1 \qquad h(-1)=$

$f'(1)=3 \qquad h'(-1)=\frac{1}{3}$

3. If $f(x)=x^3-3x^2+8x+5$ and $g(x)=f^{-1}(x)$, then $g'(5)=?$

A. 8

B. $\frac{1}{8}$

C. 1

D. $\frac{1}{53}$

E. 53

$f(0)=0+0+0+5=5$

$f'(x)=3x^2-6x+8$

$f(0)=5 \qquad g(5)=$

$f'(0)=8 \qquad g'(5)=\frac{1}{8}$

4. Find $\frac{dy}{dx}$ if $y=\ln(\sec x+\tan x)$.

A. $\frac{dy}{dx}=\sec x$

B. $\frac{dy}{dx}=\frac{1}{\sec x}$

C. $\frac{dy}{dx}=\tan x+\frac{\sec^2 x}{\tan x}$

D. $\frac{dy}{dx}=\frac{1}{\sec x+\tan x}$

E. $\frac{dy}{dx}=-\frac{1}{\sec x+\tan x}$

$\frac{dy}{dx}=\frac{1}{\sec x+\tan x}\cdot(\sec x\tan x+\sec^2 x)$

$=\frac{\sec x(\tan x+\sec x)}{\sec x+\tan x}$

5. Find $\frac{dy}{dx}$ if $y = \ln\left(\sqrt{x^2+1}\right)$. $= \ln(x^2+1)^{1/2}$

$\frac{dy}{dx} = \frac{1}{2}\left(\frac{1}{x^2+1}\right)\cdot 2x$

$= \frac{x}{x^2+1}$

A. $\frac{dy}{dx} = \frac{1}{\sqrt{x^2+1}}$

B. $\frac{dy}{dx} = \frac{2x}{\sqrt{x^2+1}}$

C. $\frac{dy}{dx} = \frac{1}{2(x^2+1)}$

D. $\frac{dy}{dx} = \frac{x}{x^2+1}$

E. $\frac{dy}{dx} = \frac{2x}{x^2+1}$

6. Find $\frac{dy}{dx}$ if $y = \ln\left(\frac{e^x}{e^x-1}\right)$ $= \ln e^x - \ln(e^x-1)$

$= x - \ln(e^x-1)$

$\frac{dy}{dx} = 1 - \frac{1}{e^x-1}\cdot e^x$

$= 1 - \frac{e^x}{e^x-1}$

$= \frac{e^x-1}{e^x-1} - \frac{e^x}{e^x-1}$

$= \frac{-1}{e^x-1}$

A. $\frac{dy}{dx} = x - \frac{e^x}{e^x-1}$

B. $\frac{dy}{dx} = \frac{1}{e^x-1}$

C. $\frac{dy}{dx} = -\frac{1}{e^x-1}$

D. $\frac{dy}{dx} = 0$

E. $\frac{dy}{dx} = \frac{e^x-2}{e^x-1}$

7. Find $\frac{dy}{dx}$ if $y = x\ln^3 x$

$\frac{dy}{dx} = (\ln^3 x)(1) + x\left(3(\ln x)^2\left(\frac{1}{x}\right)\right)$

$= \ln^3 x + 3\ln^2 x$

A. $\frac{dy}{dx} = \frac{3\ln^2 x}{x}$

B. $\frac{dy}{dx} = 3\ln^2 x$

C. $\frac{dy}{dx} = 3\ln^2 x + \ln^3 x$

D. $\frac{dy}{dx} = 3(\ln x + 1)$

E. None of these