

NON-Calculator Multiple-Choice

1. If $k(x) = 2(6^{3x})$, then $k'(x) = 2(6^{3x} \cdot \ln 6 + 3)$

A. 6^{3x+1}

B. $2e^{3x}$

C. $\frac{6^{3x}}{2}$

D. $6^{3x+1}(\ln 6)$

E. $\frac{6^{3x}}{\ln 6}$

$$= 6 \ln 6 \cdot 6^{3x}$$

$$= \ln 6 \cdot 6^{3x+1}$$

2. If $f(2) = 3$, $f'(2) = 4$, and $g(x)$ is the inverse function to $f(x)$, then the equation of the tangent line to $g(x)$ at $x = 3$ is:

A. $y - 2 = -\frac{1}{4}(x - 3)$

$$f(2) = 3 \rightarrow f'(2) = 4$$

B. $y - 2 = 4(x - 3)$

$$\underset{\nearrow}{g(3)} = 2 \leftarrow g'(3) = \frac{1}{4}$$

C. $y - 3 = -\frac{1}{4}(x - 2)$

D. $y - 2 = \frac{1}{4}(x - 3)$

E. $y - 3 = \frac{1}{4}(x - 2)$

3. If $f(x) = \ln(x + 4 + e^{-3x})$, then $f'(0) =$

A. $-\frac{2}{5}$

$$f'(x) = \frac{1}{x + 4 + e^{-3x}} \cdot 1 + e^{-3x} \cdot -3$$

B. $\frac{1}{5}$

$$= \frac{1 - 3e^{-3x}}{x + 4 + e^{-3x}}$$

C. $\frac{1}{4}$

$$f'(0) = \frac{1 - 3e^{-3(0)}}{1 + 4 + e^{-3(0)}}$$

D. $\frac{2}{5}$

$$= \frac{1 - 3}{1 + 4 + 1}$$

E. nonexistent

$$= \frac{-2}{5}$$

CALCULATOR Multiple-Choice

4. Let $f(x) = 2x^5 + 2x^3 + 2x$. If g is the inverse function of f , then $g'(6) =$

A. 18

B. 1

C. $\frac{1}{13178}$

D. $\frac{1}{6}$

E. $\frac{1}{18}$

$$6 = 2x^5 + 2x^3 + 2x$$

$$\cancel{x} \quad x = 1$$

$$f(1) = 6 \rightarrow f'(1) = 18$$

$$g(6) = 1$$

$$g'(6) = \frac{1}{18}$$

↓

Non-Calculator Free-Response

1. The twice-differential function f is defined for all real numbers and satisfies the following conditions:

$$f(0) = 2, f'(0) = -4, \text{ and } f''(0) = 3$$

- a) The function g is given by $g(x) = e^{ax} + f(x)$ for all real numbers, where a is a constant. Find $g'(0)$ and $g''(0)$ in terms of a . Show the work that leads to your answers.

$$g'(x) = e^{ax} \cdot a + f'(x)$$

$$g'(0) = e^0 \cdot a + f'(0) \quad \boxed{a-4}$$

$$g''(x) = ae^{ax} \cdot a + f''(x)$$

$$g''(0) = a^2 + 3$$

- b) The function h is given by $h(x) = \cos(kx)f(x)$ for all real numbers, where k is a constant. Find $h'(x)$ and write an equation for the line tangent to the graph of h at $x = 0$.

$$h'(x) = f(x)(-\sin(kx)) + f'(x) \cdot \cos(kx)$$

$$h(0) = \cos(k \cdot 0) f(0)$$

$$= \cos 0 f(0)$$

$$= 2$$

$$= \cancel{-f(0)} + f'(0)$$

$$= \cancel{-4} = -4$$

$$y - 2 = -4(x - 0)$$

2. Let f be the function defined by $f(x) = k\sqrt{x} - \ln x$ for $x > 0$, where k is a positive constant.

- a) Find $f'(x)$ and $f''(x)$.

$$f'(x) = k(\frac{1}{2}x^{-\frac{1}{2}}) - \frac{1}{x} \rightarrow \frac{1}{2}kx^{-\frac{1}{2}} - x^{-1}$$

$$\underline{\underline{f''(x)}} = \frac{k}{2\sqrt{x}} - \frac{1}{x^2}$$

$$f''(x) = -\frac{1}{4}kx^{-\frac{3}{2}} + \frac{1}{x^2} \rightarrow -\frac{1}{4}kx^{-\frac{3}{2}} + x^{-2}$$