DATE: _____

Unit 5 (Chapter 4): Trigonometric Functions **Pre-Calculus 2016-2017**

4.1 Angles and Their Measures

Target 5A: Describe and convert between radian and degree measure

Review of Prior Concepts

The wheels on the bus go round and round, round and round, round and round.
The wheels on the bus go round and round, all through the town.
If the radius of the wheel of the bus is 70 cm, what is the circumference of the wheel?

$C = 2\pi r$

$C = 2\pi(70) \rightarrow 140\pi = \boxed{439.823\text{ cm}}$

More Practice

Circumference

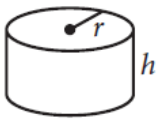
https://www.mathsisfun.com/geometry/circle.html
http://www.mathplanet.com/education/pre-algebra/more-about-equation-and-inequalities/calculating-the-circumference-of-a-circle
http://www.mathgoodies.com/lessons/vol2/circumference.html
https://www.youtube.com/watch?v=WgW_KwtBvro

SAT Connection

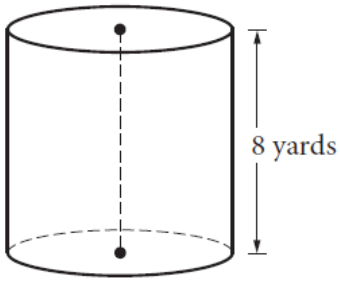
Passport to Advanced Math

14. Use structure to isolate or identify a quantity of interest in an expression

Example:



$V = \pi r^2 h$



A dairy farmer uses a storage silo that is in the shape of the right circular cylinder above. If the volume of the silo is 72π cubic yards, what is the diameter of the base of the cylinder, in yards?

$V = \pi r^2 h$

$72\pi = \pi r^2(8)$

$72\pi = 8\pi r^2$

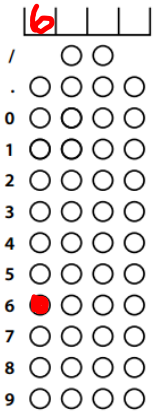
$9 = r^2$

$3 = r$

$d = 2r$

$d = 2(3)$

$d = 6$



NOTE: You may start your answers in any column, space permitting. Columns you don't need to use should be left blank.

Solution

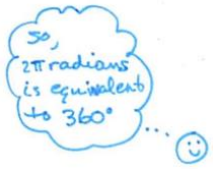
Vocabulary

- Degree - measure of an angle

  Degree of ⊙ = 360°

- Radian - unit of angular measure
  - ratio of arc length to length of radius

$$\text{Radian of } \odot = \frac{\text{Length of } \odot}{\text{Length of radius of } \odot} = \frac{2\pi r}{r} = 2\pi \text{ radians}$$



Convert from Degrees to Radians

Multiply degrees by $\frac{2\pi \text{ radians}}{360^\circ} \Rightarrow \frac{\pi \text{ radians}}{180^\circ}$

Example: Convert 36° to radians

$36^\circ \cdot \frac{\pi \text{ radians}}{180^\circ}$

$36^\circ \cdot \frac{\pi \text{ radians}}{180^\circ}$ ← degree symbols "cancel" & left with radians

$\frac{36\pi}{180}$ radians $= \frac{1}{5}\pi$ radians or $\boxed{\frac{\pi}{5} \text{ radians}}$

Convert from Radians to Degrees

Multiply radians by $\frac{360^\circ}{2\pi \text{ radians}} \Rightarrow \frac{180^\circ}{\pi \text{ radians}}$

Example: Convert $\frac{2\pi}{3}$ radians to degrees

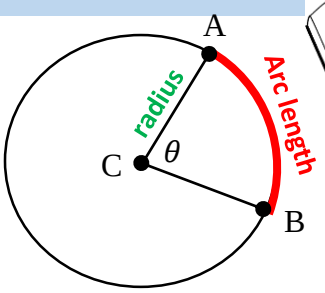
$\frac{2\pi}{3} \text{ radians} \left(\frac{180^\circ}{\pi \text{ radians}}\right)$

$\frac{2\pi}{3} \text{ radians} \left(\frac{180^\circ}{\pi \text{ radians}}\right)$ ← radian measurements "cancel" & left with degrees

$\frac{2\pi(180^\circ)}{3(\pi)} = \frac{360^\circ\pi}{3\pi} = \frac{360^\circ}{3} = \boxed{120^\circ}$

Arc Length

p. 355



$\overset{\frown}{AB} = \frac{\angle ACB}{360^\circ}$ (circumference)

$= \frac{\theta}{360^\circ}$ ($2\pi r$)

$= \frac{2\pi r\theta}{360^\circ}$

$= \frac{\pi}{180^\circ} r\theta$ where θ is measured in degrees

*What if θ is measured in radians?

$S = \frac{\pi}{180^\circ} r\theta \left(\frac{180^\circ}{\pi \text{ radians}}\right)$ ← convert to radians

$S = r\theta$ where θ is measured in radians

Examples:

radius

p.358 #28

$S = 2.5$ cm, $\theta = \frac{\pi}{3}$ radians, $r = ?$

$S = r\theta$

$2.5 = r\left(\frac{\pi}{3}\right)$

$\frac{3}{\pi} \cdot 2.5 = r\left(\frac{\pi}{3}\right) \cdot \frac{3}{\pi}$

$\frac{7.5}{\pi}$ cm $= r$

p.358 #32

$S = ?$, $r = 5$ ft, $\theta = 18^\circ$

convert to radians

$18^\circ\left(\frac{\pi \text{ radians}}{180^\circ}\right)$

$\frac{18\pi}{180}$ radians

$\frac{\pi}{10}$ radians

$S = r\theta$

$S = 5\left(\frac{\pi}{10}\right)$

$= \frac{\pi}{2}$ ft

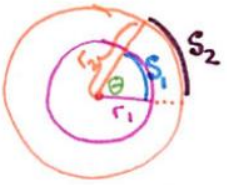
p.358 #34 concentric circles – circles w/ same center

with center

$\theta = ?$ $r_1 = 8$ km

$S_1 = 36$ km

$r_2 = ?$ $S_2 = 72$ km



$S_1 = r_1\theta$

$36 = 8\theta$

$4.5 = \theta$ radians

$S_2 = r_2\theta$

$72 = r_2(4.5)$

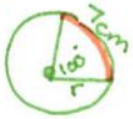
$16 \text{ km} = r_2$

$\theta = 4.5$ radians

$r_2 = 16$ km

p.358 #36

$S = 7$cm, $\theta = 100^\circ$, $r = ?$



convert 100° to radians

$100^\circ\left(\frac{\pi \text{ radians}}{180^\circ}\right)$

$\frac{100\pi}{180}$ radians

$\frac{5\pi}{9}$ radians

$S = r\theta$

$7 = r\left(\frac{5\pi}{9}\right)$

$\left(\frac{9}{5\pi}\right)7 = r\left(\frac{5\pi}{9}\right)\left(\frac{9}{5\pi}\right)$

$\frac{63}{5\pi} = r \rightarrow r = 4.011$ cm

4cm

More Practice

Converting Between Radians and Degrees

http://www.purplemath.com/modules/radians.htm
http://www.mathwarehouse.com/trigonometry/radians/convert-degee-to-radians.php
http://www.softschools.com/math/calculus/converting_between_degrees_and_radians/
https://www.youtube.com/watch?v=O3jvUZ8wvZs
https://www.youtube.com/watch?v=z0-1gBy1ykE
https://www.youtube.com/watch?v=hM7CCJbNlH8

Arc Length

http://www.regentsprep.org/regents/math/algtrig/atm1/arclengthlesson.htm
http://www.coolmath.com/reference/circles-trigonometry
https://www.khanacademy.org/math/geometry-home/cc-geometry-circles#central-angles-and-arc-length
https://www.youtube.com/watch?v=SlfRoDI3esA

Homework Assignment

p.358 #10,11,14,17,21,25-37odd

SAT Connection

Solution

The correct answer is 6. The volume of a cylinder is $\pi r^2 h$, where r is the radius of the base of the cylinder and h is the height of the cylinder. Since the storage silo is a cylinder with volume 72π cubic yards and height 8 yards, it is true that $72\pi = \pi r^2(8)$, where r is the radius of the base of the cylinder, in yards. Dividing both sides of $72\pi = \pi r^2(8)$ by 8π gives $r^2 = 9$, and so the radius of base of the cylinder is 3 yards. Therefore, the diameter of the base of the cylinder is 6 yards.